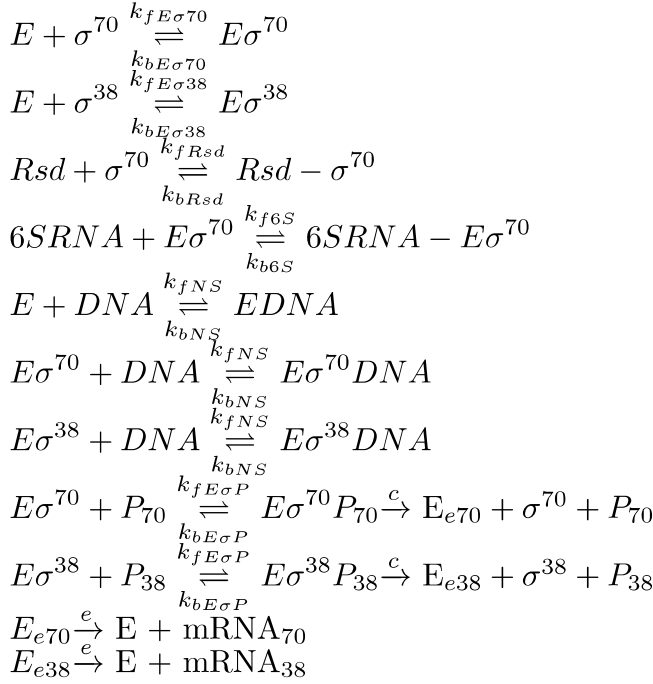


File S2: Differential Equations for Model

The model depicted in Figure 6a describes the following set of reactions:



These can be written as a system of differential equations, as given below:

$$\frac{d[E]}{dt} = -k_{fE\sigma^{70}}[E][\sigma^{70}] + k_{bE\sigma^{70}}[E\sigma^{70}] - k_{fE\sigma^{38}}[E][\sigma^{38}] + k_{bE\sigma^{38}}[E\sigma^{38}] \quad (\text{Equation S1})$$

$$\begin{aligned}
 &- k_{fNS}[E][DNA] + k_{bNS}[EDNA] + e[E_{e70}] + e[E_{e38}] \\
 \frac{d[\sigma^{70}]}{dt} &= -k_{fE\sigma^{70}}[E][\sigma^{70}] + k_{bE\sigma^{70}}[E\sigma^{70}] - k_{fRsd}[Rsd][\sigma^{70}] + k_{bRsd}[Rsd - \sigma^{70}] + c[E\sigma^{70}P_{70}] \quad (\text{Equation S2})
 \end{aligned}$$

$$\frac{d[\sigma^{38}]}{dt} = -k_{fE\sigma^{38}}[E][\sigma^{38}] + k_{bE\sigma^{38}}[E\sigma^{38}] + c[E\sigma^{38}P_{38}] \quad (\text{Equation S3})$$

$$\begin{aligned}
 \frac{d[E\sigma^{70}]}{dt} &= k_{fE\sigma^{70}}[E][\sigma^{70}] - k_{bE\sigma^{70}}[E\sigma^{70}] - k_{f6S}[6SRNA][E\sigma^{70}] + k_{b6S}[6SRNA - E\sigma^{70}] \\
 &- k_{fNS}[E\sigma^{70}][DNA] + k_{bNS}[E\sigma^{70}DNA] - k_{fE\sigma P}[E\sigma^{70}][P_{70}] + k_{bE\sigma P}[E\sigma^{70}P_{70}] \quad (\text{Equation S4})
 \end{aligned}$$

$$\begin{aligned}
 \frac{d[E\sigma^{38}]}{dt} &= k_{fE\sigma^{38}}[E][\sigma^{38}] - k_{bE\sigma^{38}}[E\sigma^{38}] - k_{fNS}[E\sigma^{38}][DNA] + k_{bNS}[E\sigma^{38}DNA] \\
 &- k_{fE\sigma P}[E\sigma^{38}][P_{38}] + k_{bE\sigma P}[E\sigma^{38}P_{38}] \quad (\text{Equation S5})
 \end{aligned}$$

$$\frac{d[Rsd]}{dt} = -k_{fRsd}[Rsd][\sigma^{70}] + k_{bRsd}[Rsd - \sigma^{70}] \quad (\text{Equation S6})$$

$$\frac{d[Rsd - \sigma^{70}]}{dt} = k_{fRsd}[Rsd][\sigma^{70}] - k_{bRsd}[Rsd - \sigma^{70}] \quad (\text{Equation S7})$$

$$\frac{d[6SRNA]}{dt} = -k_{f6S}[6SRNA][E\sigma^{70}] + k_{b6S}[6SRNA - E\sigma^{70}] \quad (\text{Equation S8})$$

$$\frac{d[6SRNA - E\sigma^{70}]}{dt} = k_{f6S}[6SRNA][E\sigma^{70}] - k_{b6S}[6SRNA - E\sigma^{70}] \quad (\text{Equation S9})$$

$$\frac{d[DNA]}{dt} = -k_{fNS}[E][DNA] + k_{bNS}[EDNA] - k_{fNS}[E\sigma^{70}][DNA] \quad (\text{Equation S10})$$

$$+ k_{bNS}[E\sigma^{70}DNA] - k_{fNS}[E\sigma^{38}][DNA] + k_{bNS}[E\sigma^{38}DNA] \\ \frac{d[EDNA]}{dt} = k_{fNS}[E][DNA] - k_{bNS}[EDNA] \quad (\text{Equation S11})$$

$$\frac{d[E\sigma^{70}DNA]}{dt} = k_{fNS}[E\sigma^{70}][DNA] - k_{bNS}[E\sigma^{70}DNA] \quad (\text{Equation S12})$$

$$\frac{d[E\sigma^{38}DNA]}{dt} = k_{fDNA}[E\sigma^{38}][DNA] - k_{bDNA}[E\sigma^{38}DNA] \quad (\text{Equation S13})$$

$$\frac{d[P_{70}]}{dt} = -k_{fE\sigma P}[E\sigma^{70}][P_{70}] + k_{bE\sigma P}[E\sigma^{70}P_{70}] + c[E\sigma^{70}P_{70}] \quad (\text{Equation S14})$$

$$\frac{d[P_{38}]}{dt} = -k_{fE\sigma P}[E\sigma^{38}][P_{38}] + k_{bE\sigma P}[E\sigma^{38}P_{38}] + c[E\sigma^{38}P_{38}] \quad (\text{Equation S15})$$

$$\frac{d[E\sigma^{70}P_{70}]}{dt} = k_{fE\sigma P}[E\sigma^{70}][P_{70}] - k_{bE\sigma P}[E\sigma^{70}P_{70}] - c[E\sigma^{70}P_{70}] \quad (\text{Equation S16})$$

$$\frac{d[E\sigma^{38}P_{38}]}{dt} = k_{fE\sigma P}[E\sigma^{38}][P_{38}] - k_{bE\sigma P}[E\sigma^{38}P_{38}] - c[E\sigma^{38}P_{38}] \quad (\text{Equation S17})$$

$$\frac{d[E_{e70}]}{dt} = c[E\sigma^{70}P_{70}] - e[E_{e70}] \quad (\text{Equation S18})$$

$$\frac{d[E_{e38}]}{dt} = c[E\sigma^{38}P_{38}] - e[E_{e38}] \quad (\text{Equation S19})$$

Where:

$$\frac{k_{bE\sigma 70}}{k_{fE\sigma 70}} = K_{E\sigma 70}$$

$$\frac{k_{bE\sigma 38}}{k_{fE\sigma 38}} = K_{E\sigma 38}$$

$$\frac{k_{bRsd}}{k_{fRsd}} = K_{Rsd}$$

$$\frac{k_{b6S}}{k_{f6S}} = K_{6S}$$

$$\frac{k_{bNS}}{k_{fNS}} = K_{NS}$$

$$\frac{k_{bE\sigma P} + c}{k_{fE\sigma P}} (\approx \frac{k_{bE\sigma P}}{k_{fE\sigma P}}) = K_{E\sigma P}$$

Rates of transcription are given by:

$$\frac{d[mRNA_{70}]}{dt} = e[E_{e70}] \quad (\text{Equation S20})$$

$$\frac{d[mRNA_{38}]}{dt} = e[E_{e38}] \quad (\text{Equation S21})$$

To obtain steady state values, the left-hand sides of equations (S1) - (S19) were set to zero. We initialized values of $E = E_{\text{total}}$, $\sigma^{70} = \sigma^{70}_{\text{total}}$, $\sigma^{38} = \sigma^{38}_{\text{total}}$, $Rsd = Rsd_{\text{total}}$, $6S\text{ RNA} = 6SRNA_{\text{total}}$, $DNA = DNA_{\text{total}}$, $P_{70} = P_{70\text{total}}$, $P_{38} = P_{38\text{total}}$ and all other variables = 0. The differential equations were solved until the system reached steady state.