# Supplementary Note 1

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| Algorithm 1. SNMF dimensionality reduction algorithm |
| Input: The matrix and the initial value of and the rank of  Output: the non-negative factorization of ,  1: repeat  2: solving in parallel;  3: solving in parallel;  4: until the relative difference in the object function between two adjacent loops is less than some threshold ( in this paper) or the maximum specified number of iterations is reached. |

The detailed process of alternate iteration algorithm used to solve mode (1) was described in algorithm 1. Here, we depict more details of step 2 in Algorithm 1. After is fixed, the optimization question (1) is equivalent to mode (2).

Notice that

where is a vector that all elements are . Therefore, solving the problem (2) is equivalent to solve n non-negativity-constrained least squares problems (3) in parallel, i.e.

It is easy to show that the problem (3) is equivalent to

where . Compared with model (3), model (4) can be solved with less computer time and less memory by using the method proposed by Lawson et al (Gentleman 1976). The rank of and the initial value of were estimated by the algorithm reported in WEDGE (Hu *et al.* 2019).

# Supplementary Note 2

The Gaussian mixture model is the sum of a set of Gaussian density functions (McLachlan *et al.* 2019), i.e.

where is a D-dimensional continuous-valued feature vector, and ( is the mixed weight). is a D-variate Gaussian density function, expressed asl

# Supplementary Note 3

**Theorem** Let be a block diagonal matrix with being a square matrix of size whose diagonal elements are and the other elements are Then is semi-definite whose eigenvalues are 0 with multiplicity , and with multiplicity Furthermore, the eigen-space associated with the eigenvalue 0 is spanned by , here is an indicator vector whose entries are 1 at indices from to , and 0 at other indices.

**Proof:**

It is easy to show that

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The first part of the conclusion follows immediately.

For the second part, since the sum of each row of is zero, clearly the indicator vector is an eigenvector associated with the eigenvalue 0, On the other hand, the multiplicity of the eigenvalue 0 is , thus span the eigenspace of the eigenvalue 0.

References

Gentleman, W. M., 1976 Solving Least Squares Problems (Charles L. Lawson and Richard J. Hanson). SIAM Rev. 18: 518–520.

Hu, Y., B. Li, N. Liu, P. Cai, F. Chen *et al.*, 2019 WEDGE: recovery of gene expression values for sparse single-cell RNA-seq datasets using matrix decomposition. bioRxiv 864488.

McLachlan, G. J., S. X. Lee, and S. I. Rathnayake, 2019 Finite Mixture Models. Annu. Rev. Stat. Its Appl. 6: 355–378.