Generalities

Working directory is notebook directory

In[∘]:=
```
SetDirectory[NotebookDirectory[]];
```

Simplifications

Function to simplify, with conditions on the parameters

In[∘]:=
```
AF[x_] :=
 Assuming[d00 > 0 && d00 < 1 && cDrive > 0 && cDrive ≤ 1 && d0D > 0 && d0D ≥ d00 && dDD > 0 &&
   pD ≥ 0 && pD ≤ 1 && Ntot ≥ 0 && diffHW ∈ Reals && dBB > 0 && dBB < 1 && h0B ≥ 0 && h0B ≤ 1 &&
   hDB ≥ 0 && hDB ≤ 1 && dBB ≥ d00 && dDD ≥ dBB && hD0 ≥ 0 && hD0 ≤ 1 && cBrake > 0 &&
   cBrake ≤ 1 && β00 > 0 && β0D > 0 && βDD > 0 && β0B > 0 && βDB > 0 && βBB > 0 &&
   b > 0 && ω00 ≥ 0 && ω0D ≥ 0 && ωDD ≥ 0 && KK > 0, FullSimplify[x]]
```

Definitions

Vector of genotype densities

In[∘]:=
$$VG = \begin{pmatrix} n00 \\ n0D \\ nDD \\ n0B \\ nDB \\ nBB \end{pmatrix};$$

Vector of fecundities per genotype

In[∘]:=
$$\beta G = \begin{pmatrix} \beta 00 \\ \beta 0D \\ \beta DD \\ \beta 0B \\ \beta DB \\ \beta BB \end{pmatrix};$$

Vector of genotype densities scaled by their fecundities

In[∘]:=
```
scaled = Diagonal[βG.Transpose[VG]];
% // MatrixForm
```

Out[∘]//MatrixForm=
$$\begin{pmatrix} n00\ \beta 00 \\ n0D\ \beta 0D \\ nDD\ \beta DD \\ n0B\ \beta 0B \\ nDB\ \beta DB \\ nBB\ \beta BB \end{pmatrix}$$

Matrix of crosses

In[•]:=
```
Crossings = Transpose[{scaled}].{scaled};
% // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} n00^2\ \beta00^2 & n00\ n0D\ \beta00\ \beta0D & n00\ nDD\ \beta00\ \beta DD & n00\ n0B\ \beta00\ \beta0B & n00\ nDB\ \beta00\ \beta DB & n00\ nBE \\ n00\ n0D\ \beta00\ \beta0D & n0D^2\ \beta0D^2 & n0D\ nDD\ \beta0D\ \beta DD & n0B\ n0D\ \beta0B\ \beta0D & n0D\ nDB\ \beta0D\ \beta DB & n0D\ nBE \\ n00\ nDD\ \beta00\ \beta DD & n0D\ nDD\ \beta0D\ \beta DD & nDD^2\ \beta DD^2 & n0B\ nDD\ \beta0B\ \beta DD & nDB\ nDD\ \beta DB\ \beta DD & nBB\ nDD \\ n00\ n0B\ \beta00\ \beta0B & n0B\ n0D\ \beta0B\ \beta0D & n0B\ nDD\ \beta0B\ \beta DD & n0B^2\ \beta0B^2 & n0B\ nDB\ \beta0B\ \beta DB & n0B\ nBE \\ n00\ nDB\ \beta00\ \beta DB & n0D\ nDB\ \beta0D\ \beta DB & nDB\ nDD\ \beta DB\ \beta DD & n0B\ nDB\ \beta0B\ \beta DB & nDB^2\ \beta DB^2 & nBB\ nDE \\ n00\ nBB\ \beta00\ \beta BB & n0D\ nBB\ \beta0D\ \beta BB & nBB\ nDD\ \beta BB\ \beta DD & n0B\ nBB\ \beta0B\ \beta BB & nBB\ nDB\ \beta BB\ \beta DB & nBB^2 \end{pmatrix}$$

Matrices for the outputs of crosses

The crosses are in the same order as the Crossings matrix (rows and columns in the order of the genotype vector);

The values are the probabilities that such a cross gives the focal genotype (focal ID in matrix name).

-> the sum of the 6 matrices is a matrix of ones. This is the law of total probabilities.

-> the sums of all elements of the homozygotes' matrices are the same, likewise for the elements of the heterozygotes' matrices. This is because there is not selection yet so alleles are interchangeable.

In[•]:=

$$\text{Cross0O} = \begin{pmatrix} 1 & 1/2 & 0 & 1/2 & 0 & 0 \\ 1/2 & 1/4 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/4 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix};$$

In[•]:=

$$\text{Cross0D} = \begin{pmatrix} 0 & 1/2 & 1 & 0 & 1/2 & 0 \\ 1/2 & 1/2 & 1/2 & 1/4 & 1/4 & 0 \\ 1 & 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 1/4 & 1/2 & 0 & 1/4 & 0 \\ 1/2 & 1/4 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix};$$

In[•]:=

$$\text{CrossDD} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/4 & 1/2 & 0 & 1/4 & 0 \\ 0 & 1/2 & 1 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/4 & 1/2 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix};$$

In[•]:=

$$\text{Cross0B} = \begin{pmatrix} 0 & 0 & 0 & 1/2 & 1/2 & 1 \\ 0 & 0 & 0 & 1/4 & 1/4 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/4 & 0 & 1/2 & 1/4 & 1/2 \\ 1/2 & 1/4 & 0 & 1/4 & 0 & 0 \\ 1 & 1/2 & 0 & 1/2 & 0 & 0 \end{pmatrix};$$

In[•]:=

$$\text{CrossDB} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 1/4 & 1/2 \\ 0 & 0 & 0 & 1/2 & 1/2 & 1 \\ 0 & 1/4 & 1/2 & 0 & 1/4 & 0 \\ 0 & 1/4 & 1/2 & 1/4 & 1/2 & 1/2 \\ 0 & 1/2 & 1 & 0 & 1/2 & 0 \end{pmatrix};$$

In[•]:=
$$\text{CrossBB} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 1/4 & 1/2 \\ 0 & 0 & 0 & 1/4 & 1/4 & 1/2 \\ 0 & 0 & 0 & 1/2 & 1/2 & 1 \end{pmatrix};$$

Check that that the sums are OK

In[•]:=
```
{Total[Total[Cross00]], Total[Total[CrossDD]], Total[Total[CrossBB]]}
{Total[Total[Cross0D]], Total[Total[Cross0B]], Total[Total[CrossDB]]}
Cross00 + CrossDD + CrossBB + Cross0D + Cross0B + CrossDB // MatrixForm
```

Out[•]= {4, 4, 4}

Out[•]= {8, 8, 8}

Out[•]//MatrixForm=
$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Check formula

In[•]:=
```
{scaled}.Cross00.scaled // MatrixForm
```

Out[•]//MatrixForm=
$$\left(n0B\ \beta 0B \left(\frac{n00\ \beta 00}{2} + \frac{n0B\ \beta 0B}{4} + \frac{n0D\ \beta 0D}{4} \right) + n0D\ \beta 0D \left(\frac{n00\ \beta 00}{2} + \frac{n0B\ \beta 0B}{4} + \frac{n0D\ \beta 0D}{4} \right) + n00\ \beta 00 \left(n00\ \beta 00 + \frac{n0B\ \beta}{2} \right.\right.$$

Matrix of gene conversion: transformations of heterozygotes into homozygotes

c_X conversion probability for type X (Drive, Brake). If conversion fails, the allele is unchanged (no resistance).

In[•]:=
$$\text{Conversion} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 - c_{Drive} & 0 & 0 & 0 & 0 \\ 0 & c_{Drive} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 - c_{Brake} & 0 \\ 0 & 0 & 0 & 0 & c_{Brake} & 1 \end{pmatrix};$$

Changing the variables : find equivalence between the two sets of variables

WT and drive only

In[•]:=
```
Solve[{Ntot == n00 + n0D + nDD, (*
    *) pD == (nDD + 1/2 n0D) / Ntot, dHW == n0D/Ntot - 2 pD (1 - pD)},
  {n00, n0D, nDD, n0B, nDB, nBB}] // FullSimplify
solCV2 = %[[1]];
```

••• Solve: Equations may not give solutions for all "solve" variables.

Out[•]=
$$\left\{\left\{n00 \to -\frac{1}{2}\,Ntot\,\left(dHW - 2\,(-1 + pD)^2\right),\right.\right.$$
$$\left.\left. n0D \to Ntot\,\left(dHW - 2\,(-1 + pD)\,pD\right),\ nDD \to -\frac{dHW\,Ntot}{2} + Ntot\,pD^2\right\}\right\}$$

Change of variables with WT, Drive and Brake

In[•]:=
```
Solve[{Ntot == n00 + n0D + nDD + n0B + nDB + nBB, (*
   *) pD == (nDD + 1/2 n0D + 1/2 nDB) / Ntot, diffHW0D == n0D/Ntot - 2 pD (1 - pD - pB), (*
   *) pB == (nBB + 1/2 n0B + 1/2 nDB) / Ntot, diffHW0B == n0B/Ntot - 2 pB (1 - pD - pB), (*
   *) diffHWDB == nDB/Ntot - 2 pD pB}, {n00, n0D, nDD, n0B, nDB, nBB}] // AF
solCV3 = %[[1]];
```

Out[•]=

$$\left\{\left\{n00 \to -\frac{1}{2}\,Ntot\,\left(diffHW0B + diffHW0D - 2\,(-1 + pB + pD)^2\right),\right.\right.$$
$$n0D \to Ntot\,(diffHW0D - 2\,pD\,(-1 + pB + pD)),$$
$$nDD \to -\frac{1}{2}\,Ntot\,(diffHW0D + diffHWDB - 2\,pD^2),$$
$$n0B \to Ntot\,(diffHW0B - 2\,pB\,(-1 + pB + pD)),\ nDB \to Ntot\,(diffHWDB + 2\,pB\,pD),$$
$$\left.\left. nBB \to -\frac{1}{2}\,Ntot\,(diffHW0B + diffHWDB - 2\,pB^2)\right\}\right\}$$

Demographic functions

Linear birth, death with density dependence

In[•]:=
```
B1[Ntot_] := b Ntot;
D1[Ntot_] := d1 + d2 Ntot + d3 Ntot^2;
```

Birth with density dependence, constant death

In[•]:=
```
demog = {Birth[Ntot] → Ntot (1 - Ntot / KK),
   Death[Ntot] → 1, Birth'[Ntot] → 1 - 2 Ntot/KK, Death'[Ntot] → 0};
```

Model with WT only

In[•]:=
```
dNWT = ω00 β00^2 Birth[Ntot] - d00 Death[Ntot] n00 /. n00 → Ntot
solWTonly = Solve[(dNWT /. demog) == 0, Ntot]
sol0 = solWTonly[[2]];
```

Out[•]= $\beta00^2\,\omega00\,Birth[Ntot] - d00\,Ntot\,Death[Ntot]$

Out[•]= $\left\{\{Ntot \to 0\},\ \left\{Ntot \to \frac{-d00\,KK + KK\,\beta00^2\,\omega00}{\beta00^2\,\omega00}\right\}\right\}$

In[•]:=
```
NWT = Ntot /. solWTonly[[2]] // AF
```

Out[•]= $KK - \frac{d00\,KK}{\beta00^2\,\omega00}$

WT viability conditions

In[•]:=
```
D[dNWT, Ntot] /. demog /. Ntot → 0
```

Out[•]= $-d00 + \beta00^2\,\omega00$

Check parameters

The parameters are standardised to yield the same equilibrium value of population size (using NWT above), which is negative in the case of an eradication drive (i.e. the actual equilibrium is then 0). So testing the three parameter combinations that we use in the paper:

In[•]:=
$$\frac{d}{\beta^2\,\omega} \text{ /. } \{d \to 1.1,\ \beta \to 1,\ \omega \to 1\}$$
$$\frac{d}{\beta^2\,\omega} \text{ /. } \{d \to 0.6,\ \beta \to 1,\ \omega \to 0.545\}$$
$$\frac{d}{\beta^2\,\omega} \text{ /. } \{d \to 0.6,\ \beta \to 0.738,\ \omega \to 1\}$$

Out[•]= 1.1

Out[•]= 1.10092

Out[•]= 1.10164

We rounded the parameter values, which is why the combinations are not *exactly* 1.1.

Parameters for replacement drive only used in the stability analysis:

In[•]:=
$$\frac{d}{\beta^2\,\omega} \text{ /. } \{d \to 0.7,\ \beta \to 1,\ \omega \to 1\}$$
$$\frac{d}{\beta^2\,\omega} \text{ /. } \{d \to 0.6,\ \beta \to 1,\ \omega \to 0.86\}$$
$$\frac{d}{\beta^2\,\omega} \text{ /. } \{d \to 0.6,\ \beta \to 0.93,\ \omega \to 1\}$$

Out[•]= 0.7

Out[•]= 0.697674

Out[•]= 0.693722

Export to C for the simulations

In[•]:=
```
chgvar = {n00 → pop[0], n0D → pop[1],
    nDD → pop[2], n0B → pop[3], nDB → pop[4], nBB → pop[5],
    β00 → beta00, β0D → beta0D, βDD → betaDD, β0B → beta0B,
    βDB → betaDB, βBB → betaBB,
    cDrive → conversionD, cBrake → conversionB,
    Ntot → popsize,
    ω00 → omega00, ω0D → omega0D,
    ωDD → omegaDD, ω0B → omega0B, ωDB → omegaDB, ωBB → omegaBB,
    Birth[Ntot] → popsize * (1 - popsize / K),
    Death[Ntot] → 1};
```

Other functions to link to C

In[•]:=
```
ComputeNeq[KK_, d00_, dDD_, dBB_, omega00_, omegaDD_, omegaBB_, beta00_,
  betaDD_, betaBB_, hD0_, hB0_, hDB_, conversionD_, conversionB_,
  N0D_, freqIntroduction_, N0B_, MAXTIME_, convType_, NREPS_] :=
```

$$\left\{\frac{-\text{d00 KK}+\text{KK beta00}^2\text{ omega00}}{\text{beta00}^2\text{ omega00}},\ \frac{-\text{dDD KK}+\text{KK betaDD}^2\text{ omegaDD}}{\text{betaDD}^2\text{ omegaDD}},\ \frac{-\text{dBB KK}+\text{KK betaBB}^2\text{ omegaBB}}{\text{betaBB}^2\text{ omegaBB}}\right\}$$

Parameters chosen in the simulations

In[•]:=
```
simKK = 25 000;
(* Does not restore fitness *)
(* Effects on d *)
ComputeNeq[simKK,
 0.6, 1.1, 1.1,
 1.0, 1.0, 1.0,
 1.0, 1.0, 1.0,
 simhD0, simhB0, simhDB,
 0.9, 0.8,
 1000, fI, 100, tfinal, typeconv, nreps]

(* Effects on omega *)
ComputeNeq[simKK,
 0.6, 0.6, 0.6,
 1.0, 0.545, 0.545,
 1.0, 1.0, 1.0,
 h, h, h,
 0.9, 0.8,
 1000, fI, 100, tfinal, typeconv, nreps]

(* Effects of beta *)
ComputeNeq[simKK,
 0.6, 0.6, 0.6,
 1.0, 1.0, 1.0,
 1.0, 0.738, 0.738,
 h, h, h,
 0.9, 0.8,
 1000, fI, 100, tfinal, typeconv, nreps]

(* Restores fitness *)
(* Effects on d *)
ComputeNeq[simKK,
 0.6, 1.1, 0.64,
 1.0, 1.0, 1.0,
 1.0, 1.0, 1.0,
```

```
 simhD0, simhB0, simhDB,
 0.9, 0.8,
 1000, fI, 100, tfinal, typeconv, nreps]

(* Effects on omega *)
ComputeNeq[simKK,
 0.6, 0.6, 0.6,
 1.0, 0.545, 0.9375,
 1.0, 1.0, 1.0,
 h, h, h,
 0.9, 0.8,
 1000, fI, 100, tfinal, typeconv, nreps]

(* Effects of beta *)
ComputeNeq[simKK,
 0.6, 0.6, 0.6,
 1.0, 1.0, 1.0,
 1.0, 0.738, 0.968246,
 h, h, h,
 0.9, 0.8,
 1000, fI, 100, tfinal, typeconv, nreps]
```

```
Out[•]= {10 000., -2500., -2500.}
Out[•]= {10 000., -2522.94, -2522.94}
Out[•]= {10 000., -2540.93, -2540.93}
Out[•]= {10 000., -2500., 9000.}
Out[•]= {10 000., -2522.94, 9000.}
Out[•]= {10 000., -2540.93, 9000.01}
```

Conversion in the germline

Model with WT and Drive

Model definition

With genotype densities

Crossing matrices : frequency at which a given cross gives the considered genotype

In[•]:= $\texttt{DCross00} = \begin{pmatrix} 1 & 1/2 & 0 \\ 1/2 & 1/4 & 0 \\ 0 & 0 & 0 \end{pmatrix};$

In[•]:=
```
DCrossOD = {{0, 1/2, 1}, {1/2, 1/2, 1/2}, {1, 1/2, 0}};
```

$$\text{DCrossOD} = \begin{pmatrix} 0 & 1/2 & 1 \\ 1/2 & 1/2 & 1/2 \\ 1 & 1/2 & 0 \end{pmatrix};$$

In[•]:=

$$\text{DCrossDD} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1/4 & 1/2 \\ 0 & 1/2 & 1 \end{pmatrix};$$

Check that they sum to 1

In[•]:=
```
DCrossOO + DCrossOD + DCrossDD
```

Out[•]=
```
{{1, 1, 1}, {1, 1, 1}, {1, 1, 1}}
```

Vector of abundances of each genotype

In[•]:=

$$\text{DVG} = \begin{pmatrix} \text{n00} \\ \text{n0D} \\ \text{nDD} \end{pmatrix};$$

Vector of fecundities

In[•]:=

$$\text{D}\beta\text{G} = \begin{pmatrix} \beta\text{00} \\ \beta\text{0D} \\ \beta\text{DD} \end{pmatrix};$$

Conversion matrix

Gene conversion takes place in the germline. With probability c_{Drive}, a OD individual will only produce D gametes.

In[•]:=

$$\text{DConversion} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 - c_{\text{Drive}} & 0 \\ 0 & c_{\text{Drive}} & 1 \end{pmatrix};$$

In[•]:=
```
DConversion.DVG // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \text{n00} \\ \text{n0D}\ (1 - c_{\text{Drive}}) \\ \text{nDD} + \text{n0D}\ c_{\text{Drive}} \end{pmatrix}$$

Abundances of the precursors of gametes of each genotype

In[•]:=
```
Dscaled = DConversion.Diagonal[DβG.Transpose[DVG]];
% // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \text{n00}\ \beta\text{00} \\ \text{n0D}\ \beta\text{0D}\ (1 - c_{\text{Drive}}) \\ \text{nDD}\ \beta\text{DD} + \text{n0D}\ \beta\text{0D}\ c_{\text{Drive}} \end{pmatrix}$$

Abundances of zygotes of each type

In[•]:=
$$\text{DVegg} = \begin{pmatrix} \{\text{Dscaled}\}.\text{DCross00}.\text{Dscaled} \\ \{\text{Dscaled}\}.\text{DCross0D}.\text{Dscaled} \\ \{\text{Dscaled}\}.\text{DCrossDD}.\text{Dscaled} \end{pmatrix} \Big/ (\text{Ntot})^2 \text{ // FullSimplify // Flatten}$$

Out[•]=
$$\left\{\frac{\left(2\,\text{n00}\,\beta\text{00} + \text{n0D}\,\beta\text{0D} - \text{n0D}\,\beta\text{0D}\,c_{\text{Drive}}\right)^2}{4\,\text{Ntot}^2},\ \frac{\left(2\,\text{n00}\,\beta\text{00} + \text{n0D}\,\beta\text{0D} - \text{n0D}\,\beta\text{0D}\,c_{\text{Drive}}\right)\left(\text{n0D}\,\beta\text{0D} + 2\,\text{nDD}\,\beta\text{DD} + \text{n0D}\,\beta\text{0D}\,c_{\text{Drive}}\right)}{2\,\text{Ntot}^2},\ \frac{\left(\text{n0D}\,\beta\text{0D} + 2\,\text{nDD}\,\beta\text{DD} + \text{n0D}\,\beta\text{0D}\,c_{\text{Drive}}\right)^2}{4\,\text{Ntot}^2}\right\}$$

Dynamics of each genotype

In[•]:=
```
DdGn00 = DVegg[[1]] ω00 Birth[Ntot] - d00 Death[Ntot] n00
DdGn0D = DVegg[[2]] ω0D Birth[Ntot] - d0D Death[Ntot] n0D
DdGnDD = DVegg[[3]] ωDD Birth[Ntot] - dDD Death[Ntot] nDD
```

Out[•]=
$$-\text{d00}\,\text{n00}\,\text{Death}[\text{Ntot}] + \frac{\omega\text{00}\,\text{Birth}[\text{Ntot}]\left(2\,\text{n00}\,\beta\text{00} + \text{n0D}\,\beta\text{0D} - \text{n0D}\,\beta\text{0D}\,c_{\text{Drive}}\right)^2}{4\,\text{Ntot}^2}$$

Out[•]=
$$-\text{d0D}\,\text{n0D}\,\text{Death}[\text{Ntot}] + \frac{1}{2\,\text{Ntot}^2}\,\omega\text{0D}\,\text{Birth}[\text{Ntot}]\left(2\,\text{n00}\,\beta\text{00} + \text{n0D}\,\beta\text{0D} - \text{n0D}\,\beta\text{0D}\,c_{\text{Drive}}\right)\left(\text{n0D}\,\beta\text{0D} + 2\,\text{nDD}\,\beta\text{DD} + \text{n0D}\,\beta\text{0D}\,c_{\text{Drive}}\right)$$

Out[•]=
$$-\text{dDD}\,\text{nDD}\,\text{Death}[\text{Ntot}] + \frac{\omega\text{DD}\,\text{Birth}[\text{Ntot}]\left(\text{n0D}\,\beta\text{0D} + 2\,\text{nDD}\,\beta\text{DD} + \text{n0D}\,\beta\text{0D}\,c_{\text{Drive}}\right)^2}{4\,\text{Ntot}^2}$$

Abundances of the alleles (/2)

In[•]:=
```
DdGn0 = DdGn00 + 1/2 DdGn0D // FullSimplify
DdGnD = DdGnDD + 1/2 DdGn0D // FullSimplify
```

Out[•]=
$$\frac{1}{4}\Bigg(-2\,(2\,\text{d00}\,\text{n00} + \text{d0D}\,\text{n0D})\,\text{Death}[\text{Ntot}] + \frac{1}{\text{Ntot}^2}\,\text{Birth}[\text{Ntot}]\left(2\,\text{n00}\,\beta\text{00} + \text{n0D}\,\beta\text{0D} - \text{n0D}\,\beta\text{0D}\,c_{\text{Drive}}\right)\left(2\,\text{n00}\,\beta\text{00}\,\omega\text{00} + 2\,\text{nDD}\,\beta\text{DD}\,\omega\text{0D} + \text{n0D}\,\beta\text{0D}\,(\omega\text{00} + \omega\text{0D}) + \text{n0D}\,\beta\text{0D}\,(-\omega\text{00} + \omega\text{0D})\,c_{\text{Drive}}\right)\Bigg)$$

Out[•]=
$$\frac{1}{4}\Bigg(-2\,(\text{d0D}\,\text{n0D} + 2\,\text{dDD}\,\text{nDD})\,\text{Death}[\text{Ntot}] + \frac{1}{\text{Ntot}^2}\,\text{Birth}[\text{Ntot}]\left(\text{n0D}\,\beta\text{0D} + 2\,\text{nDD}\,\beta\text{DD} + \text{n0D}\,\beta\text{0D}\,c_{\text{Drive}}\right)\left(2\,\text{n00}\,\beta\text{00}\,\omega\text{0D} + 2\,\text{nDD}\,\beta\text{DD}\,\omega\text{DD} + \text{n0D}\,\beta\text{0D}\,(\omega\text{0D} + \omega\text{DD}) + \text{n0D}\,\beta\text{0D}\,(-\omega\text{0D} + \omega\text{DD})\,c_{\text{Drive}}\right)\Bigg)$$

Change of variables

Total population size Ntot

In[∘]:=
```
DdGNtot = DdGn0 + DdGnD /. solCV2 // FullSimplify
```

Out[∘]=

$$
\begin{aligned}
&\frac{1}{4}\Big(2\,\mathrm{Ntot}\,\big(-2\,\mathrm{d0D}\,\mathrm{dHW}+\mathrm{d00}\,(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2)+4\,\mathrm{d0D}\,(-1+\mathrm{pD})\,\mathrm{pD}+\mathrm{dDD}\,(\mathrm{dHW}-2\,\mathrm{pD}^2)\big)\\
&\quad \mathrm{Death}[\mathrm{Ntot}]+\mathrm{Birth}[\mathrm{Ntot}]\,\Big(\big((\mathrm{dHW}-2\,(-1+\mathrm{pD})^2)\,\beta 00-\mathrm{dHW}\,\beta 0\mathrm{D}+2\,(-1+\mathrm{pD})\,\mathrm{pD}\,\beta 0\mathrm{D}\big)\\
&\qquad \Big((\mathrm{dHW}-2\,(-1+\mathrm{pD})^2)\,\beta 00\,\omega 00-(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta 0\mathrm{D}\,\omega 00-\\
&\qquad\quad 2\,\big(\mathrm{dHW}\,(\beta 0\mathrm{D}-\beta\mathrm{DD})+2\,\mathrm{pD}\,(\beta 0\mathrm{D}-\mathrm{pD}\,\beta 0\mathrm{D}+\mathrm{pD}\,\beta\mathrm{DD})\big)\,\omega 0\mathrm{D}\Big)+\\
&\qquad \big(\mathrm{dHW}\,(\beta 0\mathrm{D}-\beta\mathrm{DD})+2\,\mathrm{pD}\,(\beta 0\mathrm{D}-\mathrm{pD}\,\beta 0\mathrm{D}+\mathrm{pD}\,\beta\mathrm{DD})\big)^2\,\omega\mathrm{DD}+\\
&\qquad (\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta 0\mathrm{D}\,c_{\mathrm{Drive}}\\
&\qquad\quad \Big(2\,\Big((\mathrm{dHW}-2\,(-1+\mathrm{pD})^2)\,\beta 00\,(\omega 00-\omega 0\mathrm{D})+\mathrm{dHW}\,\big(\beta\mathrm{DD}\,(\omega 0\mathrm{D}-\omega\mathrm{DD})+\beta 0\mathrm{D}\,(-\omega 00+\omega\mathrm{DD})\big)+\\
&\qquad\qquad 2\,\mathrm{pD}\,\big((-1+\mathrm{pD})\,\beta 0\mathrm{D}\,(\omega 00-\omega\mathrm{DD})+\mathrm{pD}\,\beta\mathrm{DD}\,(-\omega 0\mathrm{D}+\omega\mathrm{DD})\big)\Big)+\\
&\qquad\quad (\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta 0\mathrm{D}\,(\omega 00-2\,\omega 0\mathrm{D}+\omega\mathrm{DD})\,c_{\mathrm{Drive}}\Big)\Big)\Big)
\end{aligned}
$$

Frequency of the drive allele

In[∘]:=
```
DdGpD = DdGnD/Ntot - pD DdGNtot/Ntot /. solCV2 // FullSimplify
```

Out[∘]=

$$
\begin{aligned}
&\frac{1}{4\,\mathrm{Ntot}}\Big(-2\,\mathrm{Ntot}\,\big(\mathrm{d0D}\,(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})-\mathrm{dDD}\,(\mathrm{dHW}-2\,\mathrm{pD}^2)\big)\,\mathrm{Death}[\mathrm{Ntot}]+\mathrm{Birth}[\mathrm{Ntot}]\\
&\quad \big(\mathrm{dHW}\,(\beta 0\mathrm{D}-\beta\mathrm{DD})+2\,\mathrm{pD}\,(\beta 0\mathrm{D}-\mathrm{pD}\,\beta 0\mathrm{D}+\mathrm{pD}\,\beta\mathrm{DD})+(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta 0\mathrm{D}\,c_{\mathrm{Drive}}\big)\\
&\quad \Big(-(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2)\,\beta 00\,\omega 0\mathrm{D}-(\mathrm{dHW}-2\,\mathrm{pD}^2)\,\beta\mathrm{DD}\,\omega\mathrm{DD}+(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\\
&\qquad \beta 0\mathrm{D}\,(\omega 0\mathrm{D}+\omega\mathrm{DD})+(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta 0\mathrm{D}\,(-\omega 0\mathrm{D}+\omega\mathrm{DD})\,c_{\mathrm{Drive}}\Big)-\\
&\quad \mathrm{pD}\,\Big(2\,\mathrm{Ntot}\,\big(-2\,\mathrm{d0D}\,\mathrm{dHW}+\mathrm{d00}\,(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2)+4\,\mathrm{d0D}\,(-1+\mathrm{pD})\,\mathrm{pD}+\mathrm{dDD}\,(\mathrm{dHW}-2\,\mathrm{pD}^2)\big)\\
&\qquad \mathrm{Death}[\mathrm{Ntot}]+\\
&\qquad \mathrm{Birth}[\mathrm{Ntot}]\,\Big(\big((\mathrm{dHW}-2\,(-1+\mathrm{pD})^2)\,\beta 00-\mathrm{dHW}\,\beta 0\mathrm{D}+2\,(-1+\mathrm{pD})\,\mathrm{pD}\,\beta 0\mathrm{D}\big)\\
&\qquad\quad \Big((\mathrm{dHW}-2\,(-1+\mathrm{pD})^2)\,\beta 00\,\omega 00+2\,\mathrm{dHW}\,\beta\mathrm{DD}\,\omega 0\mathrm{D}-\mathrm{dHW}\,\beta 0\mathrm{D}\,(\omega 00+2\,\omega 0\mathrm{D})+\\
&\qquad\qquad 2\,\mathrm{pD}\,\big(-2\,\mathrm{pD}\,\beta\mathrm{DD}\,\omega 0\mathrm{D}+(-1+\mathrm{pD})\,\beta 0\mathrm{D}\,(\omega 00+2\,\omega 0\mathrm{D})\big)\Big)+\\
&\qquad\quad \big(\mathrm{dHW}\,(\beta 0\mathrm{D}-\beta\mathrm{DD})+2\,\mathrm{pD}\,(\beta 0\mathrm{D}-\mathrm{pD}\,\beta 0\mathrm{D}+\mathrm{pD}\,\beta\mathrm{DD})\big)^2\,\omega\mathrm{DD}+\\
&\qquad\quad (\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta 0\mathrm{D}\,c_{\mathrm{Drive}}\\
&\qquad\qquad \Big(2\,\Big((\mathrm{dHW}-2\,(-1+\mathrm{pD})^2)\,\beta 00\,(\omega 00-\omega 0\mathrm{D})+\mathrm{dHW}\,\big(\beta\mathrm{DD}\,(\omega 0\mathrm{D}-\omega\mathrm{DD})+\beta 0\mathrm{D}\,(-\omega 00+\\
&\qquad\qquad\qquad \omega\mathrm{DD})\big)+2\,\mathrm{pD}\,\big((-1+\mathrm{pD})\,\beta 0\mathrm{D}\,(\omega 00-\omega\mathrm{DD})+\mathrm{pD}\,\beta\mathrm{DD}\,(-\omega 0\mathrm{D}+\omega\mathrm{DD})\big)\Big)+\\
&\qquad\qquad (\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta 0\mathrm{D}\,(\omega 00-2\,\omega 0\mathrm{D}+\omega\mathrm{DD})\,c_{\mathrm{Drive}}\Big)\Big)\Big)\Big)
\end{aligned}
$$

Deviation from HW

In[◦]:=
$$\mathrm{DdGHW} = \frac{-\mathrm{n0D}\,\mathrm{DdGNtot} + \mathrm{Ntot}\left(\mathrm{DdGn0D} + 2\,\mathrm{Ntot}\,(-1+2\,\mathrm{pD})\,\mathrm{DdGpD}\right)}{\mathrm{Ntot}^2} \ /.\ \mathrm{solCV2}$$

Out[◦]=
$$\begin{aligned}
&\frac{1}{\mathrm{Ntot}^2}\Bigg(-\frac{1}{4}\mathrm{Ntot}\,(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\\
&\quad\Big(2\,\mathrm{Ntot}\left(-2\,\mathrm{d0D}\,\mathrm{dHW}+\mathrm{d00}\left(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\right)+4\,\mathrm{d0D}\,(-1+\mathrm{pD})\,\mathrm{pD}+\mathrm{dDD}\left(\mathrm{dHW}-2\,\mathrm{pD}^2\right)\right)\\
&\qquad\mathrm{Death}[\mathrm{Ntot}]+\\
&\qquad\mathrm{Birth}[\mathrm{Ntot}]\Big(\Big(\left(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\right)\beta00-\mathrm{dHW}\,\beta0\mathrm{D}+2\,(-1+\mathrm{pD})\,\mathrm{pD}\,\beta0\mathrm{D}\Big)\\
&\qquad\quad\Big(\left(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\right)\beta00\,\omega00-(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta0\mathrm{D}\,\omega00-\\
&\qquad\quad\ 2\,(\mathrm{dHW}\,(\beta0\mathrm{D}-\beta\mathrm{DD})+2\,\mathrm{pD}\,(\beta0\mathrm{D}-\mathrm{pD}\,\beta0\mathrm{D}+\mathrm{pD}\,\beta\mathrm{DD}))\,\omega0\mathrm{D}\Big)+\\
&\qquad\quad(\mathrm{dHW}\,(\beta0\mathrm{D}-\beta\mathrm{DD})+2\,\mathrm{pD}\,(\beta0\mathrm{D}-\mathrm{pD}\,\beta0\mathrm{D}+\mathrm{pD}\,\beta\mathrm{DD}))^2\,\omega\mathrm{DD}+\\
&\qquad\quad(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta0\mathrm{D}\,c_{\mathrm{Drive}}\\
&\qquad\quad\Big(2\,\Big(\left(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\right)\beta00\,(\omega00-\omega0\mathrm{D})+\mathrm{dHW}\,(\beta\mathrm{DD}\,(\omega0\mathrm{D}-\omega\mathrm{DD})+\beta0\mathrm{D}\,(-\omega00+\\
&\qquad\quad\ \omega\mathrm{DD}))+2\,\mathrm{pD}\,((-1+\mathrm{pD})\,\beta0\mathrm{D}\,(\omega00-\omega\mathrm{DD})+\mathrm{pD}\,\beta\mathrm{DD}\,(-\omega0\mathrm{D}+\omega\mathrm{DD}))\Big)+\\
&\qquad\quad(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta0\mathrm{D}\,(\omega00-2\,\omega0\mathrm{D}+\omega\mathrm{DD})\,c_{\mathrm{Drive}}\Big)\Big)\Big)+\\
&\quad\mathrm{Ntot}\Bigg(-\mathrm{d0D}\,\mathrm{Ntot}\,(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\mathrm{Death}[\mathrm{Ntot}]+\frac{1}{2\,\mathrm{Ntot}^2}\\
&\qquad\omega0\mathrm{D}\,\mathrm{Birth}[\mathrm{Ntot}]\,\Big(-\mathrm{Ntot}\left(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\right)\beta00+\mathrm{Ntot}\,(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta0\mathrm{D}-\\
&\qquad\ \mathrm{Ntot}\,(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta0\mathrm{D}\,c_{\mathrm{Drive}}\Big)\Big(\mathrm{Ntot}\,(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta0\mathrm{D}+\\
&\qquad\ 2\left(-\frac{\mathrm{dHW}\,\mathrm{Ntot}}{2}+\mathrm{Ntot}\,\mathrm{pD}^2\right)\beta\mathrm{DD}+\mathrm{Ntot}\,(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta0\mathrm{D}\,c_{\mathrm{Drive}}\Big)+\\
&\qquad\frac{1}{2}\,(-1+2\,\mathrm{pD})\,\Big(-2\,\mathrm{Ntot}\,\left(\mathrm{d0D}\,(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})-\mathrm{dDD}\left(\mathrm{dHW}-2\,\mathrm{pD}^2\right)\right)\mathrm{Death}[\mathrm{Ntot}]+\\
&\qquad\ \mathrm{Birth}[\mathrm{Ntot}]\,\big(\mathrm{dHW}\,(\beta0\mathrm{D}-\beta\mathrm{DD})+2\,\mathrm{pD}\,(\beta0\mathrm{D}-\mathrm{pD}\,\beta0\mathrm{D}+\mathrm{pD}\,\beta\mathrm{DD})+\\
&\qquad\quad(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta0\mathrm{D}\,c_{\mathrm{Drive}}\big)\\
&\qquad\ \Big(-\left(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\right)\beta00\,\omega0\mathrm{D}-\left(\mathrm{dHW}-2\,\mathrm{pD}^2\right)\beta\mathrm{DD}\,\omega\mathrm{DD}+(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\\
&\qquad\quad\beta0\mathrm{D}\,(\omega0\mathrm{D}+\omega\mathrm{DD})+(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta0\mathrm{D}\,(-\omega0\mathrm{D}+\omega\mathrm{DD})\,c_{\mathrm{Drive}}\Big)-\\
&\qquad\ \mathrm{pD}\,\Big(2\,\mathrm{Ntot}\,\Big(-2\,\mathrm{d0D}\,\mathrm{dHW}+\mathrm{d00}\left(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\right)+4\,\mathrm{d0D}\,(-1+\mathrm{pD})\\
&\qquad\qquad\mathrm{pD}+\mathrm{dDD}\left(\mathrm{dHW}-2\,\mathrm{pD}^2\right)\Big)\,\mathrm{Death}[\mathrm{Ntot}]+\\
&\qquad\quad\mathrm{Birth}[\mathrm{Ntot}]\,\Big(\Big(\left(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\right)\beta00-\mathrm{dHW}\,\beta0\mathrm{D}+2\,(-1+\mathrm{pD})\,\mathrm{pD}\,\beta0\mathrm{D}\Big)\\
&\qquad\qquad\Big(\left(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\right)\beta00\,\omega00+2\,\mathrm{dHW}\,\beta\mathrm{DD}\,\omega0\mathrm{D}-\mathrm{dHW}\,\beta0\mathrm{D}\,(\omega00+2\,\omega0\mathrm{D})+\\
&\qquad\qquad\ 2\,\mathrm{pD}\,(-2\,\mathrm{pD}\,\beta\mathrm{DD}\,\omega0\mathrm{D}+(-1+\mathrm{pD})\,\beta0\mathrm{D}\,(\omega00+2\,\omega0\mathrm{D}))\Big)+\\
&\qquad\qquad(\mathrm{dHW}\,(\beta0\mathrm{D}-\beta\mathrm{DD})+2\,\mathrm{pD}\,(\beta0\mathrm{D}-\mathrm{pD}\,\beta0\mathrm{D}+\mathrm{pD}\,\beta\mathrm{DD}))^2\,\omega\mathrm{DD}+\\
&\qquad\qquad(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta0\mathrm{D}\,c_{\mathrm{Drive}}\,\Big(2\,\Big(\left(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\right)\beta00\,(\omega00-\omega0\mathrm{D})+\\
&\qquad\qquad\quad\mathrm{dHW}\,(\beta\mathrm{DD}\,(\omega0\mathrm{D}-\omega\mathrm{DD})+\beta0\mathrm{D}\,(-\omega00+\omega\mathrm{DD}))+\\
&\qquad\qquad\quad2\,\mathrm{pD}\,((-1+\mathrm{pD})\,\beta0\mathrm{D}\,(\omega00-\omega\mathrm{DD})+\mathrm{pD}\,\beta\mathrm{DD}\,(-\omega0\mathrm{D}+\omega\mathrm{DD}))\Big)+\\
&\qquad\qquad(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD})\,\beta0\mathrm{D}\,(\omega00-2\,\omega0\mathrm{D}+\omega\mathrm{DD})\,c_{\mathrm{Drive}}\Big)\Big)\Big)\Big)\Bigg)\Bigg)
\end{aligned}$$

Invasion conditions

Drive invasion

Generic WT equilibrium (without specifying the demographic functions)

In[•]:= `tmp = DdGNtot /. solCV2 /. {pD → 0, dHW → 0} // FullSimplify`

Out[•]= $\beta 00^2\,\omega 00\,\text{Birth}[\text{Ntot}] - \text{d00}\,\text{Ntot}\,\text{Death}[\text{Ntot}]$

In[•]:= $\text{D}\left[\text{d00 Ntot Death}[\text{Ntot}] \Big/ \left(\beta 00^2\,\omega 00\right), \text{Ntot}\right]$

Out[•]= $$\frac{\text{d00 Death}[\text{Ntot}]}{\beta 00^2\,\omega 00} + \frac{\text{d00 Ntot Death}'[\text{Ntot}]}{\beta 00^2\,\omega 00}$$

In[•]:= $$\text{Jac} = \begin{pmatrix} \text{D}[\text{DdGNtot}, \text{Ntot}] & \text{D}[\text{DdGNtot}, \text{pD}] & \text{D}[\text{DdGNtot}, \text{dHW}] \\ \text{D}[\text{DdGpD}, \text{Ntot}] & \text{D}[\text{DdGpD}, \text{pD}] & \text{D}[\text{DdGpD}, \text{dHW}] \\ \text{D}[\text{DdGHW}, \text{Ntot}] & \text{D}[\text{DdGHW}, \text{pD}] & \text{D}[\text{DdGHW}, \text{dHW}] \end{pmatrix};$$

In[•]:= $$\text{Jac0} = \text{Jac} \;/.\; \Big\{\text{pD} \to 0, \text{dHW} \to 0, \text{Birth}[\text{Ntot}] \to \text{d00 Ntot Death}[\text{Ntot}] \Big/ \left(\beta 00^2\,\omega 00\right), \text{Birth}'[\text{Ntot}] \to \frac{\text{d00 Death}[\text{Ntot}]}{\beta 00^2\,\omega 00} + \frac{\text{d00 Ntot Death}'[\text{Ntot}]}{\beta 00^2\,\omega 00}\Big\} \;//\; \text{AF}$$

Out[•]= $$\Big\{\Big\{0, -\frac{1}{\beta 00\,\omega 00} 2\,\text{Ntot Death}[\text{Ntot}] \left(\text{d00}\,\beta 00\,\omega 00 + \text{d0D}\,\beta 00\,\omega 00 - \text{d00}\,\beta 0\text{D}\,(\omega 00 + \omega 0\text{D}) + \text{d00}\,\beta 0\text{D}\,(\omega 00 - \omega 0\text{D})\,c_{\text{Drive}}\right),$$
$$\frac{1}{2\,\beta 00\,\omega 00} \text{Ntot Death}[\text{Ntot}] \left((-2\,\text{d0D} + \text{dDD})\,\beta 00\,\omega 00 + \text{d00}\,(-\beta 00\,\omega 00 - 2\,\beta\text{DD}\,\omega 0\text{D} + 2\,\beta 0\text{D}\,(\omega 00 + \omega 0\text{D})) + 2\,\text{d00}\,\beta 0\text{D}\,(-\omega 00 + \omega 0\text{D})\,c_{\text{Drive}}\right)\Big\},$$
$$\Big\{0, \frac{\text{Death}[\text{Ntot}]\left(-\text{d0D}\,\beta 00\,\omega 00 + \text{d00}\,\beta 0\text{D}\,\omega 0\text{D} + \text{d00}\,\beta 0\text{D}\,\omega 0\text{D}\,c_{\text{Drive}}\right)}{\beta 00\,\omega 00},$$
$$\frac{\text{Death}[\text{Ntot}]\left((-\text{d0D} + \text{dDD})\,\beta 00\,\omega 00 + \text{d00}\,(\beta 0\text{D} - \beta\text{DD})\,\omega 0\text{D} + \text{d00}\,\beta 0\text{D}\,\omega 0\text{D}\,c_{\text{Drive}}\right)}{2\,\beta 00\,\omega 00}\Big\},$$
$$\{0, 0, -\text{dDD Death}[\text{Ntot}]\}\Big\}$$

In[•]:= `Eigenvalues[Jac0] // AF`

Out[•]= $$\Big\{0, -\text{dDD Death}[\text{Ntot}], \frac{\text{Death}[\text{Ntot}]\left(-\text{d0D}\,\beta 00\,\omega 00 + \text{d00}\,\beta 0\text{D}\,\omega 0\text{D} + \text{d00}\,\beta 0\text{D}\,\omega 0\text{D}\,c_{\text{Drive}}\right)}{\beta 00\,\omega 00}\Big\}$$

Specifying the demographic functions
Check that sol0 is solution (WT equilibrium identified when the demographic functions were defined, above)

In[•]:= `DdGNtot /. demog /. sol0 /. pD → 0 /. dHW → 0 // AF`

Out[•]= 0

Eigenvalues

In[•]:= `ev0 = Eigenvalues[Limit[Jac /. demog, {pD → 0, dHW → 0}] /. sol0] // AF`

Out[•]= $$\Big\{-\text{dDD}, \text{d00} - \beta 00^2\,\omega 00, \frac{-\text{d0D}\,\beta 00\,\omega 00 + \text{d00}\,\beta 0\text{D}\,\omega 0\text{D} + \text{d00}\,\beta 0\text{D}\,\omega 0\text{D}\,c_{\text{Drive}}}{\beta 00\,\omega 00}\Big\}$$

The first one is negative, because dDD is greater than (or equal to) d00,
the second one is about 00 only,
so the one that determines invasion of the drive is the third one.

Write the key eigenvalue to export it into R

```
In[•]:= evG0 = ev0[[3]];
        % /. chgvar // CForm
```

```
Out[•]//CForm= (-(beta00*d0D*omega00) + beta0D*d00*omega0D + beta0D*conversionD*d00*omega0D)/
```

WT invasion -- eradication drive

Demographic equilibrium with drive only (specifying the demographic functions)

```
In[•]:= DdGNtot /. {pD → 1, dHW → 0}
        % /. demog
        solDonly = Solve[% == 0, Ntot]
```

Out[•]= $\frac{1}{4}\left(4\,\beta DD^2\,\omega DD\,\mathrm{Birth}[\mathrm{Ntot}] - 4\,\mathrm{dDD}\,\mathrm{Ntot}\,\mathrm{Death}[\mathrm{Ntot}]\right)$

Out[•]= $\frac{1}{4}\left(-4\,\mathrm{dDD}\,\mathrm{Ntot} + 4\,\mathrm{Ntot}\left(1-\frac{\mathrm{Ntot}}{\mathrm{KK}}\right)\beta DD^2\,\omega DD\right)$

Out[•]= $\left\{\{\mathrm{Ntot}\to 0\},\ \left\{\mathrm{Ntot}\to\frac{-\mathrm{dDD}\,\mathrm{KK}+\mathrm{KK}\,\beta DD^2\,\omega DD}{\beta DD^2\,\omega DD}\right\}\right\}$

We choose conditions such that there is population extinction with the drive (Ntot -> 0):
Check stability conditions in this case (i.e. what are the conditions on the parameters to indeed have an eradication drive)

```
In[•]:= D[Evaluate[DdGNtot /. {pD → 1, dHW → 0} /. demog], Ntot] /. Ntot → 0
        AF[Reduce[% < 0]]
```

Out[•]= $\frac{1}{4}\left(-4\,\mathrm{dDD} + 4\,\beta DD^2\,\omega DD\right)$

Out[•]= $\sqrt{\frac{\mathrm{dDD}}{\omega DD}} > \beta DD\ ||\ \omega DD \le 0$

Eigenvalues for the stability of the drive-only equilibrium

```
In[•]:= evD = Eigenvalues[Limit[Jac /. demog, {Ntot → 0, pD → 1, dHW → 0}]] // AF
```

Out[•]= $\{-\mathrm{dDD} + \beta DD^2\,\omega DD,\ -\mathrm{d00} + \mathrm{dDD} - \beta DD^2\,\omega DD,$
$-\mathrm{d0D} + \mathrm{dDD} + \beta 0D\,\beta DD\,\omega 0D - \beta DD^2\,\omega DD - \beta 0D\,\beta DD\,\omega 0D\,c_{\mathrm{Drive}}\}$

Rewrite them for export in R

```
In[•]:= evGD1 = evD[[2]];
        % /. chgvar // CForm
```

```
Out[•]//CForm= -d00 + dDD - Power(betaDD,2)*omegaDD
```

```
In[•]:= evGD2 = evD[[3]];
        % /. chgvar // CForm
```

```
Out[•]//CForm= -d0D + dDD + beta0D*betaDD*omega0D - beta0D*betaDD*conversionD*omega0D - Power
```

WT invasion -- replacement drive

Here we assume a replacement drive, so use the other equilibrium

In[•]:=
```
solDonly[[2]]
```

Out[•]= $\left\{\text{Ntot} \to \frac{-\text{dDD KK} + \text{KK}\,\beta\text{DD}^2\,\omega\text{DD}}{\beta\text{DD}^2\,\omega\text{DD}}\right\}$

Check stability conditions in this case

In[•]:=
```
D[Evaluate[DdGNtot /. {pD → 1, dHW → 0} /. demog], Ntot] /. solDonly[[2]]
AF[Reduce[% < 0]]
```

Out[•]= $\frac{1}{4}\left(-4\,\text{dDD} - \frac{4\left(-\text{dDD KK} + \text{KK}\,\beta\text{DD}^2\,\omega\text{DD}\right)}{\text{KK}} + 4\,\beta\text{DD}^2\,\omega\text{DD}\left(1 - \frac{-\text{dDD KK} + \text{KK}\,\beta\text{DD}^2\,\omega\text{DD}}{\text{KK}\,\beta\text{DD}^2\,\omega\text{DD}}\right)\right)$

Out[•]= $\beta\text{DD} > \sqrt{\frac{\text{dDD}}{\omega\text{DD}}}$

Eigenvalues for the stability of the drive-only equilibrium

In[•]:=
```
evDr = Eigenvalues[
   Limit[Jac /. demog, {Ntot → (Ntot /. solDonly[[2]]), pD → 1, dHW → 0}]] // AF
```

Out[•]= $\left\{-\text{d00},\ \text{dDD} - \beta\text{DD}^2\,\omega\text{DD},\ \frac{\text{dDD}\,\beta\text{0D}\,\omega\text{0D} - \text{d0D}\,\beta\text{DD}\,\omega\text{DD} - \text{dDD}\,\beta\text{0D}\,\omega\text{0D}\,c_{\text{Drive}}}{\beta\text{DD}\,\omega\text{DD}}\right\}$

Plot the types of equilibria -- eradication drive

Function to standardize the way the figures are plotted

In[•]:=
```
InvasionPic[P1_, P2_] :=
 Show[P1, P2, FrameLabel → {"cDrive", "h0 D"}, BaseStyle → {FontSize → 13},
  FrameStyle → Directive[FontColor → Black], ImageSize → 200]
```

Rename the key eigenvalues

In[•]:=
```
EV0 = ev0[[3]];
```

In[•]:=
```
EVD1 = evD[[3]];
EVD2 = evD[[2]];
```

Adult death only

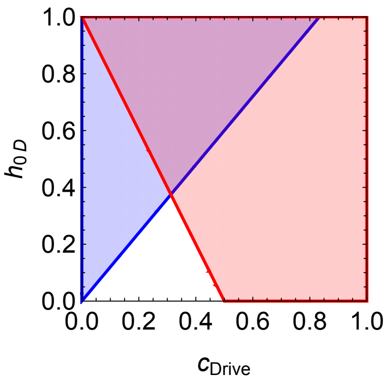
Here we assume that the effect of the drive (/brake) is only on adult mortality; all other parameters are the same as WT individuals

In[•]:=
```
ADO =
  {ω00 → 1, ω0D → 1, ωDD → 1, β00 → 1, β0D → 1, βDD → 1, d0D → d00 (1 - h0D) + dDD h0D};
```

```
In[•]:= prms = {d00 → 0.6, dDD → 1.1};
sol0 /. ADO /. prms
ADOP1 = RegionPlot[Evaluate[(EV0 /. ADO /. prms)] < 0,
   {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
   PlotStyle → RGBColor[0, 0, 1, 0.2], BoundaryStyle → Blue];
ADOP2 = RegionPlot[Evaluate[(EVD1 /. ADO /. prms)] < 0 &&
    Evaluate[(EVD2 /. ADO /. prms)] < 0,
   {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
   PlotStyle → RGBColor[1, 0, 0, 0.2], BoundaryStyle → Red];
ADG0 = InvasionPic[ADOP1, ADOP2]
Export["../pics/invasionE_Gd.pdf", ADG0]
```

```
Out[•]= {Ntot → 0.4 KK}
```

Out[•]=




```
Out[•]= ../pics/invasionE_Gd.pdf
```

Legend:

WT - only equilibrium is locally stable in the region left of the blue curve,

Drive - only equilibrium is locally stable in the region right of the red curve.

Purple: both equilibria are locally stable; bistability

White: none of the two boundary equilibria is locally stable: coexistence of WT and drive

Interior equilibrium for the parameters tested in R

```
In[•]:= neqs =
  Flatten[Evaluate[{DdGNtot == 0, DdGpD == 0, DdGHW == 0} /. demog /. ADO /. prms /.
      KK → 25 000 /. h0D → 1 /. cDrive → 0.6]];
NSolve[neqs, {Ntot, pD, dHW}, Reals]
```

```
Out[•]= {{dHW → 0, pD → 1., Ntot → -2500.},
  {dHW → -0.0330327, pD → 0.195531, Ntot → 5796.09}, {dHW → 0, pD → 0, Ntot → 10 000.}}
```

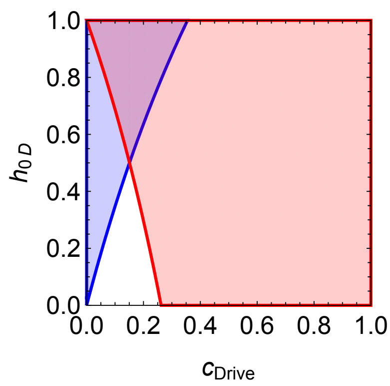
Fecundity only

Here we assume that only fecundity is affected by the drive

```
In[•]:= FO = {ω00 → 1, ω0D → 1, ωDD → 1, d00 → 0.6,
   d0D → 0.6, dDD → 0.6, β0D → β00 (1 - h0D) + βDD h0D};
```

In[∘]:=
```
prms = {β00 → 1, βDD → 0.738};
FOP1 = RegionPlot[(EV0 /. FO /. prms) < 0,
   {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
   PlotStyle → RGBColor[0, 0, 1, 0.2], BoundaryStyle → Blue];
FOP2 = RegionPlot[Evaluate[(EVD1 /. FO /. prms)] < 0 &&
     Evaluate[(EVD2 /. FO /. prms)] < 0,
    {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
    PlotStyle → RGBColor[1, 0, 0, 0.2], BoundaryStyle → Red];
FG0 = InvasionPic[FOP1, FOP2]
Export["../pics/invasionE_Gbeta.pdf", FG0]
```

Out[∘]=



Out[∘]= ../pics/invasionE_Gbeta.pdf

Legend : see above (Adult death only)

Interior equilibrium for the parameters tested in R

In[∘]:=
```
neqs = Flatten[Evaluate[
     {DdGNtot == 0, DdGpD == 0, DdGHW == 0} /. demog /. FO /. prms /. KK → 25 000 /.
       h0D → 1 /. cDrive → 0.3]];
NSolve[neqs, {Ntot, pD, dHW}, Reals]
```

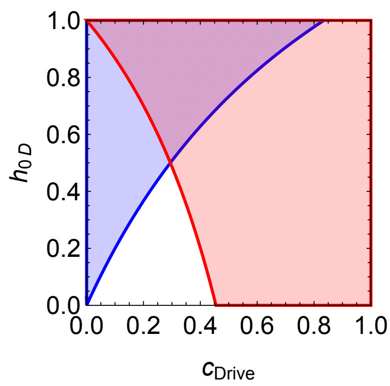
Out[∘]=
```
{{dHW → 0, pD → 1., Ntot → -2540.93}, {dHW → 0, pD → 0.154962, Ntot → 7472.42},
 {dHW → 0, pD → 0, Ntot → 10 000.}, {Ntot → 10 000., pD → 0, dHW → 0}}
```

Zygote viability only

In[∘]:=
```
ZO = {d00 → 0.6, d0D → 0.6, dDD → 0.6,
    β00 → 1, β0D → 1, βDD → 1, ω0D → ω00 (1 - h0D) + ωDD h0D};
```

In[•]:=
```
prms = {ω00 → 1, ωDD → 0.545};
ZOP1 = RegionPlot[(EV0 /. ZO /. prms) < 0,
   {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
   PlotStyle → RGBColor[0, 0, 1, 0.2], BoundaryStyle → Blue];
ZOP2 = RegionPlot[Evaluate[(EVD1 /. ZO /. prms)] < 0 &&
    Evaluate[(EVD2 /. ZO /. prms)] < 0,
   {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
   PlotStyle → RGBColor[1, 0, 0, 0.2], BoundaryStyle → Red];
OmegaG0 = InvasionPic[ZOP1, ZOP2]
Export["../pics/invasionE_Gomega.pdf", OmegaG0]
```

Out[•]=



Out[•]= ../pics/invasionE_Gomega.pdf

Legend : see above (Adult death only)

Interior equilibrium for the parameters tested in R

In[•]:=
```
neqs = Flatten[Evaluate[
    {DdGNtot == 0, DdGpD == 0, DdGHW == 0} /. demog /. ZO /. prms /. KK → 25 000 /.
     h0D → 1 /. cDrive → 0.5]];
NSolve[neqs, {Ntot, pD, dHW}, Reals]
```

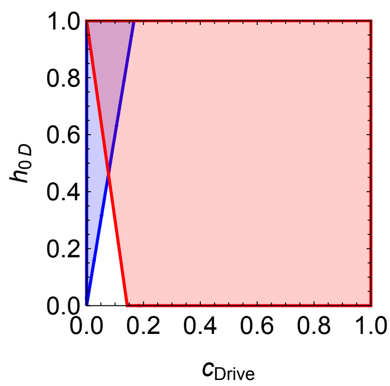
Out[•]=
```
{{dHW → 0, pD → 1., Ntot → -2522.94},
 {Ntot → 3819.56, pD → 0.308668, dHW → -0.0570609}, {dHW → 0, pD → 0, Ntot → 10 000.}}
```

Plot the types of equilibria -- replacement drive

In[•]:=
```
EVD1 =.; EVD2 =.;
EVD1r = evDr[[3]];
EVD2r = evDr[[2]];
```

Adult mortality only

In[∘]:=
```
prms = {d00 → 0.6, dDD → 0.7};
ADOP1 = RegionPlot[Evaluate[(EV0 /. ADO /. prms)] < 0,
   {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
   PlotStyle → RGBColor[0, 0, 1, 0.2], BoundaryStyle → Blue];
ADOP2 = RegionPlot[Evaluate[(EVD1r /. ADO /. prms)] < 0 &&
    Evaluate[(EVD2r /. ADO /. prms)] < 0,
   {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
   PlotStyle → RGBColor[1, 0, 0, 0.2], BoundaryStyle → Red];
ADG0 = InvasionPic[ADOP1, ADOP2]
Export["../pics/invasionR_Gd.pdf", ADG0]
```

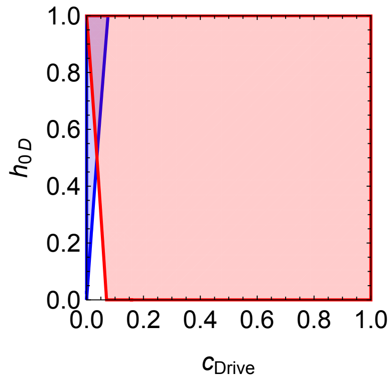
Out[∘]=


Out[∘]= ../pics/invasionR_Gd.pdf

Fecundity only

In[•]:=
```
prms = {β00 → 1, βDD → 0.93};
FOP1 = RegionPlot[(EV0 /. FO /. prms) < 0,
  {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
  PlotStyle → RGBColor[0, 0, 1, 0.2], BoundaryStyle → Blue];
FOP2 = RegionPlot[Evaluate[(EVD1r /. FO /. prms)] < 0 &&
    Evaluate[(EVD2r /. FO /. prms)] < 0,
   {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
   PlotStyle → RGBColor[1, 0, 0, 0.2], BoundaryStyle → Red];
FG0 = InvasionPic[FOP1, FOP2]
Export["../pics/invasionR_Gbeta.pdf", FG0]
```

Out[•]=

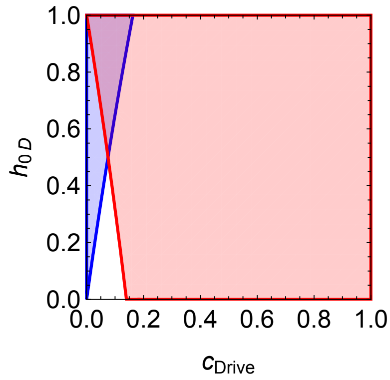


Out[•]= ../pics/invasionR_Gbeta.pdf

Zygote viability only

In[•]:=
```
prms = {ω00 → 1, ωDD → 0.86};
ZOP1 = RegionPlot[(EV0 /. ZO /. prms) < 0,
    {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
    PlotStyle → RGBColor[0, 0, 1, 0.2], BoundaryStyle → Blue];
ZOP2 = RegionPlot[Evaluate[(EVD1r /. ZO /. prms)] < 0 &&
     Evaluate[(EVD2r /. ZO /. prms)] < 0,
    {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
    PlotStyle → RGBColor[1, 0, 0, 0.2], BoundaryStyle → Red];
OmegaG0 = InvasionPic[ZOP1, ZOP2]
Export["../pics/invasionR_Gomega.pdf", OmegaG0]
```

Out[•]=



Out[•]= ../pics/invasionR_Gomega.pdf

Model with brake

Model definition

Original variables

Gamete precursors - taking gene conversion into account

In[•]:=
```
scaled = Conversion.Diagonal[βG.Transpose[VG]];
% // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} n00\ \beta00 \\ n0D\ \beta0D\ (1 - c_{Drive}) \\ nDD\ \beta DD + n0D\ \beta0D\ c_{Drive} \\ n0B\ \beta0B \\ nDB\ \beta DB\ (1 - c_{Brake}) \\ nBB\ \beta BB + nDB\ \beta DB\ c_{Brake} \end{pmatrix}$$

Different types of zygotes

In[•]:=
```
Vegg = ( {scaled}.Cross00.scaled
         {scaled}.Cross0D.scaled
         {scaled}.CrossDD.scaled
         {scaled}.Cross0B.scaled
         {scaled}.CrossDB.scaled
         {scaled}.CrossBB.scaled ) / (Ntot)^2 // FullSimplify // Flatten;

Vegg // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \frac{(2\,n00\,\beta00+n0B\,\beta0B+n0D\,\beta0D-n0D\,\beta0D\,c_{Drive})^2}{4\,Ntot^2} \\ \frac{(2\,n00\,\beta00+n0B\,\beta0B+n0D\,\beta0D-n0D\,\beta0D\,c_{Drive})\,(n0D\,\beta0D+nDB\,\beta DB+2\,nDD\,\beta DD-nDB\,\beta DB\,c_{Brake}+n0D\,\beta0D\,c_{Drive})}{2\,Ntot^2} \\ \frac{(n0D\,\beta0D+nDB\,\beta DB+2\,nDD\,\beta DD-nDB\,\beta DB\,c_{Brake}+n0D\,\beta0D\,c_{Drive})^2}{4\,Ntot^2} \\ \frac{(n0B\,\beta0B+2\,nBB\,\beta BB+nDB\,\beta DB+nDB\,\beta DB\,c_{Brake})\,(2\,n00\,\beta00+n0B\,\beta0B+n0D\,\beta0D-n0D\,\beta0D\,c_{Drive})}{2\,Ntot^2} \\ \frac{(n0B\,\beta0B+2\,nBB\,\beta BB+nDB\,\beta DB+nDB\,\beta DB\,c_{Brake})\,(n0D\,\beta0D+nDB\,\beta DB+2\,nDD\,\beta DD-nDB\,\beta DB\,c_{Brake}+n0D\,\beta0D\,c_{Drive})}{2\,Ntot^2} \\ \frac{(n0B\,\beta0B+2\,nBB\,\beta BB+nDB\,\beta DB+nDB\,\beta DB\,c_{Brake})^2}{4\,Ntot^2} \end{pmatrix}$$

Dynamics of the different genotypes

In[•]:=
```
dGn00 = Vegg[[1]] ω00 Birth[Ntot] - d00 Death[Ntot] n00
dGn0D = Vegg[[2]] ω0D Birth[Ntot] - d0D Death[Ntot] n0D
dGnDD = Vegg[[3]] ωDD Birth[Ntot] - dDD Death[Ntot] nDD
dGn0B = Vegg[[4]] ω0B Birth[Ntot] - d0B Death[Ntot] n0B
dGnDB = Vegg[[5]] ωDB Birth[Ntot] - dDB Death[Ntot] nDB
dGnBB = Vegg[[6]] ωBB Birth[Ntot] - dBB Death[Ntot] nBB
```

Out[•]= $$-d00\,n00\,Death[Ntot] + \frac{\omega00\,Birth[Ntot]\,(2\,n00\,\beta00+n0B\,\beta0B+n0D\,\beta0D-n0D\,\beta0D\,c_{Drive})^2}{4\,Ntot^2}$$

Out[•]= $$-d0D\,n0D\,Death[Ntot] + \frac{1}{2\,Ntot^2}\,\omega0D\,Birth[Ntot]\,(2\,n00\,\beta00+n0B\,\beta0B+n0D\,\beta0D-n0D\,\beta0D\,c_{Drive})\,(n0D\,\beta0D+nDB\,\beta DB+2\,nDD\,\beta DD-nDB\,\beta DB\,c_{Brake}+n0D\,\beta0D\,c_{Drive})$$

Out[•]= $$-dDD\,nDD\,Death[Ntot] + \frac{\omega DD\,Birth[Ntot]\,(n0D\,\beta0D+nDB\,\beta DB+2\,nDD\,\beta DD-nDB\,\beta DB\,c_{Brake}+n0D\,\beta0D\,c_{Drive})^2}{4\,Ntot^2}$$

Out[•]= $$-d0B\,n0B\,Death[Ntot] + \frac{1}{2\,Ntot^2}\,\omega0B\,Birth[Ntot]\,(n0B\,\beta0B+2\,nBB\,\beta BB+nDB\,\beta DB+nDB\,\beta DB\,c_{Brake})\,(2\,n00\,\beta00+n0B\,\beta0B+n0D\,\beta0D-n0D\,\beta0D\,c_{Drive})$$

Out[•]= $$-dDB\,nDB\,Death[Ntot] + \frac{1}{2\,Ntot^2}\,\omega DB\,Birth[Ntot]\,(n0B\,\beta0B+2\,nBB\,\beta BB+nDB\,\beta DB+nDB\,\beta DB\,c_{Brake})\,(n0D\,\beta0D+nDB\,\beta DB+2\,nDD\,\beta DD-nDB\,\beta DB\,c_{Brake}+n0D\,\beta0D\,c_{Drive})$$

Out[•]= $$-dBB\,nBB\,Death[Ntot] + \frac{\omega BB\,Birth[Ntot]\,(n0B\,\beta0B+2\,nBB\,\beta BB+nDB\,\beta DB+nDB\,\beta DB\,c_{Brake})^2}{4\,Ntot^2}$$

Export for C

In[•]:= dGn00 /. chgvar // CForm

Out[•]//CForm= -(d00*pop(0)) + (omega00*(1 - popsize/K)*Power(2*beta00*pop(0) + beta0D*pop(1)

In[•]:= dGn0D /. chgvar // CForm

Out[•]//CForm= -(d0D*pop(1)) + (omega0D*(1 - popsize/K)*(2*beta00*pop(0) + beta0D*pop(1) - be
(beta0D*pop(1) + beta0D*conversionD*pop(1) + 2*betaDD*pop(2) + betaDB*pop

In[•]:= dGnDD /. chgvar // CForm

Out[•]//CForm= -(dDD*pop(2)) + (omegaDD*(1 - popsize/K)*Power(beta0D*pop(1) + beta0D*conversi

In[•]:= dGn0B /. chgvar // CForm

Out[•]//CForm= -(d0B*pop(3)) + (omega0B*(1 - popsize/K)*(2*beta00*pop(0) + beta0D*pop(1) - be
(beta0B*pop(3) + betaDB*pop(4) + betaDB*conversionB*pop(4) + 2*betaBB*pop

In[•]:= dGnDB /. chgvar // CForm

Out[•]//CForm= -(dDB*pop(4)) + (omegaDB*(1 - popsize/K)*(beta0D*pop(1) + beta0D*conversionD*p
(beta0B*pop(3) + betaDB*pop(4) + betaDB*conversionB*pop(4) + 2*betaBB*pop

In[•]:= dGnBB /. chgvar // CForm

Out[•]//CForm= -(dBB*pop(5)) + (omegaBB*(1 - popsize/K)*Power(beta0B*pop(3) + betaDB*pop(4) +

Abundance of the alleles (/2)

In[•]:= dGn0 = dGn00 + 1 / 2 dGn0D + 1 / 2 dGn0B // FullSimplify;
dGnD = dGnDD + 1 / 2 dGn0D + 1 / 2 dGnDB // FullSimplify;
dGnB = dGnBB + 1 / 2 dGn0B + 1 / 2 dGnDB // FullSimplify;

Changing the variables

In[•]:= dGNtot = dGn0 + dGnD + dGnB /. solCV3;

In[•]:= $dGpD = \frac{dGnD}{Ntot} - \frac{(nDD + 1/2\ n0D + 1/2\ n0B)\ dGNtot}{Ntot^2}$ /. solCV3 ;

$dGpB = \frac{dGnB}{Ntot} - \frac{(nBB + 1/2\ n0B + 1/2\ nDB)\ dGNtot}{Ntot^2}$ /. solCV3;

In[•]:= $dGHW0D = \frac{-n0D\ dGNtot + Ntot\ (dGn0D + 2\ Ntot\ (pD\ dGpB + (-1 + pB + 2\ pD)\ dGpD))}{Ntot^2}$ /. solCV3;

$dGHWDB = \frac{Ntot\ dGnDB - nDB\ dGNtot}{Ntot^2} - 2\ (pD\ dGpB + pB\ dGpD)$ /. solCV3;

dGHW0B =

$\frac{-n0B\ dGNtot + Ntot\ (dGn0B + 2\ Ntot\ (pB\ dGpD + (-1 + pD + 2\ pB)\ dGpB))}{Ntot^2}$ /. solCV3;

Invasion conditions

Stability of the WT only equilibrium

Jacobian matrix

```
In[•]:= JacG =
  D[dGNtot, Ntot]  D[dGNtot, pD]  D[dGNtot, pB]  D[dGNtot, diffHW0D]  D[dGNtot, diffH
   D[dGpD, Ntot]    D[dGpD, pD]    D[dGpD, pB]    D[dGpD, diffHW0D]    D[dGpD, diffHW
   D[dGpB, Ntot]    D[dGpB, pD]    D[dGpB, pB]    D[dGpB, diffHW0D]    D[dGpB, diffHW
  D[dGHW0D, Ntot]  D[dGHW0D, pD]  D[dGHW0D, pB]  D[dGHW0D, diffHW0D]  D[dGHW0D, diffH
  D[dGHW0B, Ntot]  D[dGHW0B, pD]  D[dGHW0B, pB]  D[dGHW0B, diffHW0D]  D[dGHW0B, diffH
  D[dGHWDB, Ntot]  D[dGHWDB, pD]  D[dGHWDB, pB]  D[dGHWDB, diffHW0D]  D[dGHWDB, diffH
```

```
Out[•]=
{{⋯1⋯}, ⋯4⋯, {-2 (⋯1⋯)/Ntot^3 + ⋯1⋯/Ntot^2 -
   2 (pD (-1/(4 Ntot^2) (-2 (-dBB Ntot (diffHW0B + diffHWDB - 2 pB^2) + dDB Ntot
            (diffHWDB + 2 pB pD) + d0B Ntot (diffHW0B - 2 pB (-1 + pB + pD)))
         Death[Ntot] + Birth[Ntot] (⋯1⋯) (-Ntot (⋯1⋯) β00 ω0B + ⋯11⋯ + ⋯1⋯) / Ntot^2) -
       (1/2 (-diffHW0B - diffHWDB + 2 pB^2) + 1/2 (⋯1⋯) + 1/2 (diffHW0B - ⋯1⋯)) (⋯1⋯) / Ntot^2 +
       2 (⋯1⋯) (⋯1⋯) / Ntot^3 + -2 (⋯1⋯) Death[Ntot] + ⋯7⋯ / (4 Ntot) -
       1/Ntot^2 (-1/2 Ntot (diffHW0B + diffHWDB - 2 pB^2) +
          1/2 Ntot (diffHWDB + 2 pB pD) + 1/2 Ntot (diffHW0B - 2 pB (-1 + pB + pD)))
        (-(d0D (diffHW0D - ⋯1⋯) - ⋯1⋯) ⋯1⋯ +
          ⋯8⋯ + ⋯1⋯)) +
     pB (⋯1⋯)), ⋯4⋯, ⋯1⋯/Ntot^2 - 2 ⋯1⋯}}

large output   show less   show more   show all   set size limit...
```

Jacobian matrix, specifying the demographic functions

```
In[•]:= JacGdemog =
  D[dGNtot /. demog, Ntot]  D[dGNtot /. demog, pD]  D[dGNtot /. demog, pB]  D[dGNtot /
   D[dGpD /. demog, Ntot]    D[dGpD /. demog, pD]    D[dGpD /. demog, pB]    D[dGpD /.
   D[dGpB /. demog, Ntot]    D[dGpB /. demog, pD]    D[dGpB /. demog, pB]    D[dGpB /.
  D[dGHW0D /. demog, Ntot]  D[dGHW0D /. demog, pD]  D[dGHW0D /. demog, pB]  D[dGHW0D /
  D[dGHW0B /. demog, Ntot]  D[dGHW0B /. demog, pD]  D[dGHW0B /. demog, pB]  D[dGHW0B /
  D[dGHWDB /. demog, Ntot]  D[dGHWDB /. demog, pD]  D[dGHWDB /. demog, pB]  D[dGHWDB /
;
```

WT-only stability

In[•]:=
```
evGWT =
  Eigenvalues[AF[JacG /. {pD → 0, pB → 0, diffHW0B → 0, diffHW0D → 0, diffHWDB → 0} /.
      demog /. sol0]] // AF
```

Out[•]=
$$\left\{-\text{dBB}, -\text{dDB}, -\text{dDD}, \text{d00} - \beta 00^2\, \omega 00, -\text{d0B} + \frac{\text{d00}\ \beta\text{0B}\ \omega\text{0B}}{\beta 00\ \omega 00}, \frac{-\text{d0D}\ \beta 00\ \omega 00 + \text{d00}\ \beta\text{0D}\ \omega\text{0D} + \text{d00}\ \beta\text{0D}\ \omega\text{0D}\ c_{\text{Drive}}}{\beta 00\ \omega 00}\right\}$$

Comparison to the value that was previously found -> same

In[•]:=
```
evGWT[[6]] - EV0 // AF
```

Out[•]= 0

Conversion in the zygote

Model with WT and Drive

Model definition

Original variables

Matrices of crosses in the reduced model

In[•]:=
$$\text{DCross00} = \begin{pmatrix} 1 & 1/2 & 0 \\ 1/2 & 1/4 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

In[•]:=
$$\text{DCross0D} = \begin{pmatrix} 0 & 1/2 & 1 \\ 1/2 & 1/2 & 1/2 \\ 1 & 1/2 & 0 \end{pmatrix};$$

In[•]:=
$$\text{DCrossDD} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1/4 & 1/2 \\ 0 & 1/2 & 1 \end{pmatrix};$$

Vector of genotypes

In[•]:=
$$\text{DV}_G = \begin{pmatrix} \text{n00} \\ \text{n0D} \\ \text{nDD} \end{pmatrix};$$

Vector of fecundities

In[•]:=
$$\text{D}\beta_G = \begin{pmatrix} \beta 00 \\ \beta\text{0D} \\ \beta\text{DD} \end{pmatrix};$$

Genotypes scaled by their fecundities

In[•]:=
```
DZscaled = Diagonal[Dβ_G.Transpose[DV_G]];
% // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} n00\ \beta00 \\ n0D\ \beta0D \\ nDD\ \beta DD \end{pmatrix}$$

Frequencies of the different types of zygotes (pre gene conversion)

In[•]:=

$$DV_{egg} = \begin{pmatrix} \{DZscaled\}.DCross00.DZscaled \\ \{DZscaled\}.DCross0D.DZscaled \\ \{DZscaled\}.DCrossDD.DZscaled \end{pmatrix} \Big/ (Ntot)^2 \text{ // FullSimplify}$$

Out[•]=

$$\left\{\left\{\left\{\frac{(2\ n00\ \beta00 + n0D\ \beta0D)^2}{4\ Ntot^2}\right\}\right\}, \left\{\left\{\frac{(2\ n00\ \beta00 + n0D\ \beta0D)\ (n0D\ \beta0D + 2\ nDD\ \beta DD)}{2\ Ntot^2}\right\}\right\}, \left\{\left\{\frac{(n0D\ \beta0D + 2\ nDD\ \beta DD)^2}{4\ Ntot^2}\right\}\right\}\right\}$$

Matrix of gene conversion

In[•]:=

$$DM_{Conversion} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 - c_{Drive} & 0 \\ 0 & c_{Drive} & 1 \end{pmatrix};$$

Frequencies of the zygotes, post gene conversion

In[•]:=

$$DNV_G = DM_{Conversion}.DV_{egg} \text{ // Flatten}$$

Out[•]=

$$\left\{\frac{(2\ n00\ \beta00 + n0D\ \beta0D)^2}{4\ Ntot^2}, \frac{(2\ n00\ \beta00 + n0D\ \beta0D)\ (n0D\ \beta0D + 2\ nDD\ \beta DD)\ (1 - c_{Drive})}{2\ Ntot^2}, \frac{(n0D\ \beta0D + 2\ nDD\ \beta DD)^2}{4\ Ntot^2} + \frac{(2\ n00\ \beta00 + n0D\ \beta0D)\ (n0D\ \beta0D + 2\ nDD\ \beta DD)\ c_{Drive}}{2\ Ntot^2}\right\}$$

Dynamics of the different types of genotypes

In[•]:=

$$DdZn00 = DNV_G[[1]]\ \omega00\ Birth[Ntot] - d00\ Death[Ntot]\ n00$$
$$DdZn0D = DNV_G[[2]]\ \omega0D\ Birth[Ntot] - d0D\ Death[Ntot]\ n0D$$
$$DdZnDD = DNV_G[[3]]\ \omega DD\ Birth[Ntot] - dDD\ Death[Ntot]\ nDD$$

Out[•]=

$$\frac{(2\ n00\ \beta00 + n0D\ \beta0D)^2\ \omega00\ Birth[Ntot]}{4\ Ntot^2} - d00\ n00\ Death[Ntot]$$

Out[•]=

$$-d0D\ n0D\ Death[Ntot] + \frac{(2\ n00\ \beta00 + n0D\ \beta0D)\ (n0D\ \beta0D + 2\ nDD\ \beta DD)\ \omega0D\ Birth[Ntot]\ (1 - c_{Drive})}{2\ Ntot^2}$$

Out[•]=

$$-dDD\ nDD\ Death[Ntot] + \omega DD\ Birth[Ntot] \left(\frac{(n0D\ \beta0D + 2\ nDD\ \beta DD)^2}{4\ Ntot^2} + \frac{(2\ n00\ \beta00 + n0D\ \beta0D)\ (n0D\ \beta0D + 2\ nDD\ \beta DD)\ c_{Drive}}{2\ Ntot^2}\right)$$

With other variables

Allelic densities

In[∘]:=
```
DdZn0 = DdZn00 + 1/2 DdZn0D /. solCV2;
DdZnD = DdZnDD + 1/2 DdZn0D /. solCV2 // FullSimplify
```

Out[∘]=

$$\frac{1}{4}\Big(-2\,\mathrm{Ntot}\,\big((\mathrm{d0D}-\mathrm{dDD})\,\mathrm{dHW}+2\,\mathrm{d0D}\,\mathrm{pD}+2\,(-\mathrm{d0D}+\mathrm{dDD})\,\mathrm{pD}^2\big)\,\mathrm{Death}[\mathrm{Ntot}]+\big(\mathrm{dHW}\,(\beta\mathrm{0D}-\beta\mathrm{DD})+2\,\mathrm{pD}\,(\beta\mathrm{0D}-\mathrm{pD}\,\beta\mathrm{0D}+\mathrm{pD}\,\beta\mathrm{DD})\big)\,\mathrm{Birth}[\mathrm{Ntot}]\Big(-\big(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\big)\,\beta\mathrm{00}\,\omega\mathrm{0D}+\big(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD}\big)\,\beta\mathrm{0D}\,\omega\mathrm{0D}+\mathrm{dHW}\,(\beta\mathrm{0D}-\beta\mathrm{DD})\,\omega\mathrm{DD}+2\,\mathrm{pD}\,(\beta\mathrm{0D}-\mathrm{pD}\,\beta\mathrm{0D}+\mathrm{pD}\,\beta\mathrm{DD})\,\omega\mathrm{DD}+\Big(\big(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\big)\,\beta\mathrm{00}-\mathrm{dHW}\,\beta\mathrm{0D}+2\,(-1+\mathrm{pD})\,\mathrm{pD}\,\beta\mathrm{0D}\Big)\,(\omega\mathrm{0D}-2\,\omega\mathrm{DD})\,c_{\mathrm{Drive}}\Big)\Big)$$

Total population size

In[∘]:=
```
DdZNtot = DdZn0 + DdZnD;
```

Proportion of drive

In[∘]:=

$$\mathrm{DdZpD} = \frac{\mathrm{DdZnD}}{\mathrm{Ntot}} - \mathrm{pD}\,\frac{\mathrm{DdZNtot}}{\mathrm{Ntot}}\ \text{// FullSimplify}$$

Out[∘]=

$$\frac{1}{4\,\mathrm{Ntot}}\Bigg(-2\,\mathrm{Ntot}\,\big((\mathrm{d0D}-\mathrm{dDD})\,\mathrm{dHW}+2\,\mathrm{d0D}\,\mathrm{pD}+2\,(-\mathrm{d0D}+\mathrm{dDD})\,\mathrm{pD}^2\big)\,\mathrm{Death}[\mathrm{Ntot}]+\big(\mathrm{dHW}\,(\beta\mathrm{0D}-\beta\mathrm{DD})+2\,\mathrm{pD}\,(\beta\mathrm{0D}-\mathrm{pD}\,\beta\mathrm{0D}+\mathrm{pD}\,\beta\mathrm{DD})\big)\,\mathrm{Birth}[\mathrm{Ntot}]\Big(-\big(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\big)\,\beta\mathrm{00}\,\omega\mathrm{0D}+\big(\mathrm{dHW}-2\,(-1+\mathrm{pD})\,\mathrm{pD}\big)\,\beta\mathrm{0D}\,\omega\mathrm{0D}+\mathrm{dHW}\,(\beta\mathrm{0D}-\beta\mathrm{DD})\,\omega\mathrm{DD}+2\,\mathrm{pD}\,(\beta\mathrm{0D}-\mathrm{pD}\,\beta\mathrm{0D}+\mathrm{pD}\,\beta\mathrm{DD})\,\omega\mathrm{DD}+\Big(\big(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\big)\,\beta\mathrm{00}-\mathrm{dHW}\,\beta\mathrm{0D}+2\,(-1+\mathrm{pD})\,\mathrm{pD}\,\beta\mathrm{0D}\Big)\,(\omega\mathrm{0D}-2\,\omega\mathrm{DD})\,c_{\mathrm{Drive}}\Big)+\mathrm{pD}\,\bigg(-2\,\mathrm{Ntot}\,\Big(-2\,\mathrm{d0D}\,\mathrm{dHW}+\mathrm{d00}\,\big(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\big)+4\,\mathrm{d0D}\,(-1+\mathrm{pD})\,\mathrm{pD}+\mathrm{dDD}\,\big(\mathrm{dHW}-2\,\mathrm{pD}^2\big)\Big)\,\mathrm{Death}[\mathrm{Ntot}]-\mathrm{Birth}[\mathrm{Ntot}]\,\bigg(\Big(\big(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\big)\,\beta\mathrm{00}-\mathrm{dHW}\,\beta\mathrm{0D}+2\,(-1+\mathrm{pD})\,\mathrm{pD}\,\beta\mathrm{0D}\Big)\Big(\Big(\big(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\big)\,\beta\mathrm{00}-\mathrm{dHW}\,\beta\mathrm{0D}+2\,(-1+\mathrm{pD})\,\mathrm{pD}\,\beta\mathrm{0D}\Big)\,\omega\mathrm{00}-2\,\big(\mathrm{dHW}\,(\beta\mathrm{0D}-\beta\mathrm{DD})+2\,\mathrm{pD}\,(\beta\mathrm{0D}-\mathrm{pD}\,\beta\mathrm{0D}+\mathrm{pD}\,\beta\mathrm{DD})\big)\,\omega\mathrm{0D}\Big)+\big(\mathrm{dHW}\,(\beta\mathrm{0D}-\beta\mathrm{DD})+2\,\mathrm{pD}\,(\beta\mathrm{0D}-\mathrm{pD}\,\beta\mathrm{0D}+\mathrm{pD}\,\beta\mathrm{DD})\big)^2\,\omega\mathrm{DD}+2\,\Big(\big(\mathrm{dHW}-2\,(-1+\mathrm{pD})^2\big)\,\beta\mathrm{00}-\mathrm{dHW}\,\beta\mathrm{0D}+2\,(-1+\mathrm{pD})\,\mathrm{pD}\,\beta\mathrm{0D}\Big)\big(\mathrm{dHW}\,(\beta\mathrm{0D}-\beta\mathrm{DD})+2\,\mathrm{pD}\,(\beta\mathrm{0D}-\mathrm{pD}\,\beta\mathrm{0D}+\mathrm{pD}\,\beta\mathrm{DD})\big)\,(\omega\mathrm{0D}-\omega\mathrm{DD})\,c_{\mathrm{Drive}}\bigg)\bigg)\Bigg)$$

Deviation from Hardy - Weinberg

In[∘]:=

$$\mathrm{DdZHW} = \frac{-\mathrm{n0D}\,\mathrm{DdZNtot}+\mathrm{Ntot}\,\big(\mathrm{DdZn0D}+2\,\mathrm{Ntot}\,(-1+2\,\mathrm{pD})\,\mathrm{DdZpD}\big)}{\mathrm{Ntot}^2}\ \text{/. solCV2}$$

Out[•]=

$$
\begin{aligned}
&\frac{1}{\text{Ntot}^2}\Bigg(-\text{Ntot}\,(\text{dHW}-2\,(-1+\text{pD})\,\text{pD})\\
&\quad\Bigg(\frac{\left(-\text{Ntot}\,\left(\text{dHW}-2\,(-1+\text{pD})^2\right)\,\beta 00+\text{Ntot}\,(\text{dHW}-2\,(-1+\text{pD})\,\text{pD})\,\beta 0\text{D}\right)^2\,\omega 00\,\text{Birth}[\text{Ntot}]}{4\,\text{Ntot}^2}+\\
&\qquad\frac{1}{2}\,\text{d00}\,\text{Ntot}\,\left(\text{dHW}-2\,(-1+\text{pD})^2\right)\,\text{Death}[\text{Ntot}]+\\
&\qquad\frac{1}{2}\,\Bigg(-\text{d0D}\,\text{Ntot}\,(\text{dHW}-2\,(-1+\text{pD})\,\text{pD})\,\text{Death}[\text{Ntot}]+\\
&\qquad\quad\frac{1}{2\,\text{Ntot}^2}\,\left(-\text{Ntot}\,\left(\text{dHW}-2\,(-1+\text{pD})^2\right)\,\beta 00+\text{Ntot}\,(\text{dHW}-2\,(-1+\text{pD})\,\text{pD})\,\beta 0\text{D}\right)\\
&\qquad\quad\left(\text{Ntot}\,(\text{dHW}-2\,(-1+\text{pD})\,\text{pD})\,\beta 0\text{D}+2\,\left(-\frac{\text{dHW}\,\text{Ntot}}{2}+\text{Ntot}\,\text{pD}^2\right)\,\beta\text{DD}\right)\\
&\qquad\quad\omega 0\text{D}\,\text{Birth}[\text{Ntot}]\,(1-c_{\text{Drive}})\Bigg)+\\
&\qquad\frac{1}{4}\,\Bigg(-2\,\text{Ntot}\,\left((\text{d0D}-\text{dDD})\,\text{dHW}+2\,\text{d0D}\,\text{pD}+2\,(-\text{d0D}+\text{dDD})\,\text{pD}^2\right)\,\text{Death}[\text{Ntot}]+\\
&\qquad\quad(\text{dHW}\,(\beta 0\text{D}-\beta\text{DD})+2\,\text{pD}\,(\beta 0\text{D}-\text{pD}\,\beta 0\text{D}+\text{pD}\,\beta\text{DD}))\,\text{Birth}[\text{Ntot}]\\
&\qquad\quad\Big(-\left(\text{dHW}-2\,(-1+\text{pD})^2\right)\,\beta 00\,\omega 0\text{D}+(\text{dHW}-2\,(-1+\text{pD})\,\text{pD})\,\beta 0\text{D}\,\omega 0\text{D}+\\
&\qquad\qquad\text{dHW}\,(\beta 0\text{D}-\beta\text{DD})\,\omega\text{DD}+2\,\text{pD}\,(\beta 0\text{D}-\text{pD}\,\beta 0\text{D}+\text{pD}\,\beta\text{DD})\,\omega\text{DD}+\Big(\left(\text{dHW}-2\,(-1+\text{pD})^2\right)\\
&\qquad\qquad\quad\beta 00-\text{dHW}\,\beta 0\text{D}+2\,(-1+\text{pD})\,\text{pD}\,\beta 0\text{D}\Big)\,(\omega 0\text{D}-2\,\omega\text{DD})\,c_{\text{Drive}}\Big)\Bigg)\Bigg)+\\
&\quad\text{Ntot}\,\Bigg(-\text{d0D}\,\text{Ntot}\,(\text{dHW}-2\,(-1+\text{pD})\,\text{pD})\,\text{Death}[\text{Ntot}]+\frac{1}{2\,\text{Ntot}^2}\\
&\qquad\left(-\text{Ntot}\,\left(\text{dHW}-2\,(-1+\text{pD})^2\right)\,\beta 00+\text{Ntot}\,(\text{dHW}-2\,(-1+\text{pD})\,\text{pD})\,\beta 0\text{D}\right)\\
&\qquad\left(\text{Ntot}\,(\text{dHW}-2\,(-1+\text{pD})\,\text{pD})\,\beta 0\text{D}+2\,\left(-\frac{\text{dHW}\,\text{Ntot}}{2}+\text{Ntot}\,\text{pD}^2\right)\,\beta\text{DD}\right)\\
&\qquad\omega 0\text{D}\,\text{Birth}[\text{Ntot}]\,(1-c_{\text{Drive}})+\frac{1}{2}\,(-1+2\,\text{pD})\\
&\qquad\Big(-2\,\text{Ntot}\,\left((\text{d0D}-\text{dDD})\,\text{dHW}+2\,\text{d0D}\,\text{pD}+2\,(-\text{d0D}+\text{dDD})\,\text{pD}^2\right)\,\text{Death}[\text{Ntot}]+\\
&\qquad\quad(\text{dHW}\,(\beta 0\text{D}-\beta\text{DD})+2\,\text{pD}\,(\beta 0\text{D}-\text{pD}\,\beta 0\text{D}+\text{pD}\,\beta\text{DD}))\,\text{Birth}[\text{Ntot}]\\
&\qquad\quad\Big(-\left(\text{dHW}-2\,(-1+\text{pD})^2\right)\,\beta 00\,\omega 0\text{D}+(\text{dHW}-2\,(-1+\text{pD})\,\text{pD})\,\beta 0\text{D}\,\omega 0\text{D}+\\
&\qquad\qquad\text{dHW}\,(\beta 0\text{D}-\beta\text{DD})\,\omega\text{DD}+2\,\text{pD}\,(\beta 0\text{D}-\text{pD}\,\beta 0\text{D}+\text{pD}\,\beta\text{DD})\,\omega\text{DD}+\Big(\left(\text{dHW}-2\,(-1+\text{pD})^2\right)\\
&\qquad\qquad\quad\beta 00-\text{dHW}\,\beta 0\text{D}+2\,(-1+\text{pD})\,\text{pD}\,\beta 0\text{D}\Big)\,(\omega 0\text{D}-2\,\omega\text{DD})\,c_{\text{Drive}}\Big)+\\
&\qquad\text{pD}\,\Big(-2\,\text{Ntot}\,\Big(-2\,\text{d0D}\,\text{dHW}+\text{d00}\,\left(\text{dHW}-2\,(-1+\text{pD})^2\right)+4\,\text{d0D}\,(-1+\text{pD})\\
&\qquad\qquad\quad\text{pD}+\text{dDD}\,\left(\text{dHW}-2\,\text{pD}^2\right)\Big)\,\text{Death}[\text{Ntot}]-\\
&\qquad\qquad\text{Birth}[\text{Ntot}]\,\Big(\Big(\left(\left(\text{dHW}-2\,(-1+\text{pD})^2\right)\,\beta 00-\text{dHW}\,\beta 0\text{D}+2\,(-1+\text{pD})\,\text{pD}\,\beta 0\text{D}\right)\\
&\qquad\qquad\quad\Big(\left(\left(\text{dHW}-2\,(-1+\text{pD})^2\right)\,\beta 00-\text{dHW}\,\beta 0\text{D}+2\,(-1+\text{pD})\,\text{pD}\,\beta 0\text{D}\right)\,\omega 00-\\
&\qquad\qquad\quad\quad 2\,(\text{dHW}\,(\beta 0\text{D}-\beta\text{DD})+2\,\text{pD}\,(\beta 0\text{D}-\text{pD}\,\beta 0\text{D}+\text{pD}\,\beta\text{DD}))\,\omega 0\text{D}\Big)+\\
&\qquad\qquad\quad(\text{dHW}\,(\beta 0\text{D}-\beta\text{DD})+2\,\text{pD}\,(\beta 0\text{D}-\text{pD}\,\beta 0\text{D}+\text{pD}\,\beta\text{DD}))^2\,\omega\text{DD}+2\\
&\qquad\qquad\quad\left(\left(\text{dHW}-2\,(-1+\text{pD})^2\right)\,\beta 00-\text{dHW}\,\beta 0\text{D}+2\,(-1+\text{pD})\,\text{pD}\,\beta 0\text{D}\right)\\
&\qquad\qquad\quad(\text{dHW}\,(\beta 0\text{D}-\beta\text{DD})+2\,\text{pD}\,(\beta 0\text{D}-\text{pD}\,\beta 0\text{D}+\text{pD}\,\beta\text{DD}))\,(\omega 0\text{D}-\omega\text{DD})\,c_{\text{Drive}}\Big)\Big)\Big)\Bigg)\Bigg)
\end{aligned}
$$

Stability of the equilibria

Jacobian matrix

In[∘]:=
```
Jac2 = ( D[DdZNtot, Ntot]  D[DdZNtot, pD]  D[DdZHW, dHW]
         D[DdZpD, Ntot]    D[DdZpD, pD]    D[DdZpD, dHW]
         D[DdZHW, Ntot]    D[DdZHW, pD]    D[DdZHW, dHW] );
```

Stability of the WT only equilibrium

Generic version without specifying the demographic function

In[∘]:=
```
Jac2 /. {pD → 0, dHW → 0, Birth[Ntot] → d00 Death[Ntot] Ntot / β00^2} // FullSimplify;
ev2invD = Eigenvalues[%] // FullSimplify
```

Out[∘]=
$$\Big\{-\frac{1}{2\,\beta00^5}\Big(\beta00^4\,\mathrm{Death}[\mathrm{Ntot}]\,\big(\beta00\,(\mathrm{d0D}+\mathrm{dDD}+2\,\mathrm{d00}\,(-1+\omega00))-\mathrm{d00}\,\beta0D\,\omega0D+\mathrm{d00}\,(\beta0D\,\omega0D-2\,\beta DD\,\omega DD)\,c_{\mathrm{Drive}}\big)+\sqrt{\beta00^8\,\mathrm{Death}[\mathrm{Ntot}]^2\,\big((-\mathrm{d0D}\,\beta00+\mathrm{dDD}\,\beta00+\mathrm{d00}\,\beta0D\,\omega0D)^2+\mathrm{d00}\,c_{\mathrm{Drive}}\,(-2\,\beta0D\,\omega0D\,(-\mathrm{d0D}\,\beta00+\mathrm{dDD}\,\beta00+\mathrm{d00}\,\beta0D\,\omega0D)+4\,\beta DD\,(\mathrm{d0D}\,\beta00-\mathrm{dDD}\,\beta00+\mathrm{d00}\,\beta0D\,\omega0D)\,\omega DD+\mathrm{d00}\,(\beta0D\,\omega0D-2\,\beta DD\,\omega DD)^2\,c_{\mathrm{Drive}})\big)}\Big),$$
$$\frac{1}{2\,\beta00^5}\Big(-\beta00^4\,\mathrm{Death}[\mathrm{Ntot}]\,\big(\beta00\,(\mathrm{d0D}+\mathrm{dDD}+2\,\mathrm{d00}\,(-1+\omega00))-\mathrm{d00}\,\beta0D\,\omega0D+\mathrm{d00}\,(\beta0D\,\omega0D-2\,\beta DD\,\omega DD)\,c_{\mathrm{Drive}}\big)+\sqrt{\beta00^8\,\mathrm{Death}[\mathrm{Ntot}]^2\,\big((-\mathrm{d0D}\,\beta00+\mathrm{dDD}\,\beta00+\mathrm{d00}\,\beta0D\,\omega0D)^2+\mathrm{d00}\,c_{\mathrm{Drive}}\,(-2\,\beta0D\,\omega0D\,(-\mathrm{d0D}\,\beta00+\mathrm{dDD}\,\beta00+\mathrm{d00}\,\beta0D\,\omega0D)+4\,\beta DD\,(\mathrm{d0D}\,\beta00-\mathrm{dDD}\,\beta00+\mathrm{d00}\,\beta0D\,\omega0D)\,\omega DD+\mathrm{d00}\,(\beta0D\,\omega0D-2\,\beta DD\,\omega DD)^2\,c_{\mathrm{Drive}})\big)}\Big),$$
$$\beta00^2\,\omega00\,\mathrm{Birth}'[\mathrm{Ntot}]-\mathrm{d00}\,(\mathrm{Death}[\mathrm{Ntot}]+\mathrm{Ntot}\,\mathrm{Death}'[\mathrm{Ntot}])\Big\}$$

In[∘]:=
```
evZ0 = Eigenvalues[Jac2 /. demog /. {pD → 0, dHW → 0} /. sol0 // Simplify] // AF
```

Out[∘]=
$$\Big\{\mathrm{d00}-\beta00^2\,\omega00,$$
$$-\frac{1}{2\,\beta00\,\omega00}\Big(\mathrm{d0D}\,\beta00\,\omega00+\mathrm{dDD}\,\beta00\,\omega00-\mathrm{d00}\,\beta0D\,\omega0D+\mathrm{d00}\,\beta0D\,\omega0D\,c_{\mathrm{Drive}}-2\,\mathrm{d00}\,\beta DD\,\omega DD\,c_{\mathrm{Drive}}+\sqrt{\big((\mathrm{d0D}+\mathrm{dDD})\,\beta00\,\omega00-\mathrm{d00}\,\beta0D\,\omega0D+\mathrm{d00}\,(\beta0D\,\omega0D-2\,\beta DD\,\omega DD)\,c_{\mathrm{Drive}}\big)^2+4\,\beta00\,\omega00\,\big(-\mathrm{d0D}\,\mathrm{dDD}\,\beta00\,\omega00+\mathrm{d00}\,\mathrm{dDD}\,\beta0D\,\omega0D+\mathrm{d00}\,(-\mathrm{dDD}\,\beta0D\,\omega0D+2\,\mathrm{d0D}\,\beta DD\,\omega DD)\,c_{\mathrm{Drive}}\big)}\Big),$$
$$\frac{1}{2\,\beta00\,\omega00}\Big(-\mathrm{d0D}\,\beta00\,\omega00-\mathrm{dDD}\,\beta00\,\omega00+\mathrm{d00}\,\beta0D\,\omega0D-\mathrm{d00}\,\beta0D\,\omega0D\,c_{\mathrm{Drive}}+2\,\mathrm{d00}\,\beta DD\,\omega DD\,c_{\mathrm{Drive}}+\sqrt{\big((\mathrm{d0D}+\mathrm{dDD})\,\beta00\,\omega00-\mathrm{d00}\,\beta0D\,\omega0D+\mathrm{d00}\,(\beta0D\,\omega0D-2\,\beta DD\,\omega DD)\,c_{\mathrm{Drive}}\big)^2+4\,\beta00\,\omega00\,\big(-\mathrm{d0D}\,\mathrm{dDD}\,\beta00\,\omega00+\mathrm{d00}\,\mathrm{dDD}\,\beta0D\,\omega0D+\mathrm{d00}\,(-\mathrm{dDD}\,\beta0D\,\omega0D+2\,\mathrm{d0D}\,\beta DD\,\omega DD)\,c_{\mathrm{Drive}}\big)}\Big)\Big\}$$

Export

In[•]:=
```
evZ0[[3]] /. chgvar // CForm
```

Out[•]//CForm=
```
(-(beta00*d0D*omega00) - beta00*dDD*omega00 + beta0D*d00*omega0D - beta0D*conve
      Sqrt(Power(beta00*(d0D + dDD)*omega00 - beta0D*d00*omega0D + conversionD*(
        4*beta00*omega00*(-(beta00*d0D*dDD*omega00) + beta0D*d00*dDD*omega0D + (
    (2.*beta00*omega00)
```

Stability of the drive only equilibrium -- eradication drive

In[•]:=
```
evZD = Eigenvalues[Limit[Jac2 /. demog, {pD → 1, dHW → 0, Ntot → 0}]] // AF
```

Out[•]=
$\{-dDD + \beta DD^2\, \omega DD,\ -d00 + dDD - \beta DD^2\, \omega DD,$
$-d0D + dDD + \beta 0D\, \beta DD\, \omega 0D - \beta DD^2\, \omega DD - \beta 0D\, \beta DD\, \omega 0D\, c_{Drive}\}$

Export

In[•]:=
```
evZD[[2]] /. chgvar // CForm
```

Out[•]//CForm=
```
-d00 + dDD - Power(betaDD,2)*omegaDD
```

In[•]:=
```
evZD[[3]] /. chgvar // CForm
```

Out[•]//CForm=
```
-d0D + dDD + beta0D*betaDD*omega0D - beta0D*betaDD*conversionD*omega0D - Power
```

Stability of the drive only equilibrium -- replacement drive

In[•]:=
```
solZDonly = Solve[(DdZNtot /. demog /. pD → 1 /. dHW → 0) == 0, Ntot]
```

Out[•]=
$\left\{\{Ntot \to 0\},\ \left\{Ntot \to \frac{-dDD\, KK + KK\, \beta DD^2\, \omega DD}{\beta DD^2\, \omega DD}\right\}\right\}$

In[•]:=
```
evZDr = Eigenvalues[
   Limit[Jac2 /. demog, {pD → 1, dHW → 0, Ntot → (Ntot /. solZDonly[[2]])}]] // AF
```

Out[•]=
$\left\{-d00,\ dDD - \beta DD^2\, \omega DD,\ \frac{dDD\, \beta 0D\, \omega 0D - d0D\, \beta DD\, \omega DD - dDD\, \beta 0D\, \omega 0D\, c_{Drive}}{\beta DD\, \omega DD}\right\}$

Plot the types of equilibria -- eradication drive

Rename the key eigenvalues

In[•]:=
```
EV0 = evZ0[[3]];
```

In[•]:=
```
EVD1 = evZD[[3]];
EVD2 = evZD[[2]];
```

Adult death only

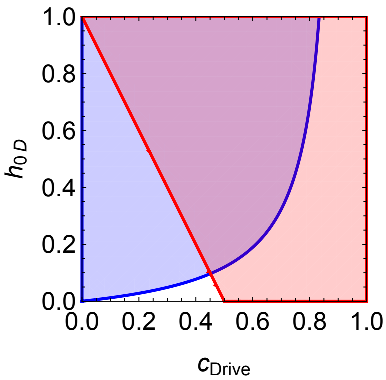
Here we assume that the effect of the drive (/brake) is only on adult mortality; all other parameters are the same as WT individuals

In[•]:=
```
ADO =
  {ω00 → 1, ω0D → 1, ωDD → 1, β00 → 1, β0D → 1, βDD → 1, d0D → d00 (1 - h0D) + dDD h0D};
```

In[∘]:=
```
prms = {d00 → 0.6, dDD → 1.1};
sol0 /. ADO /. prms
ADOP1 = RegionPlot[Evaluate[(EV0 /. ADO /. prms)] < 0,
   {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
   PlotStyle → RGBColor[0, 0, 1, 0.2], BoundaryStyle → Blue];
ADOP2 = RegionPlot[Evaluate[(EVD1 /. ADO /. prms)] < 0 &&
    Evaluate[(EVD2 /. ADO /. prms)] < 0,
   {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
   PlotStyle → RGBColor[1, 0, 0, 0.2], BoundaryStyle → Red];
AZ0 = InvasionPic[ADOP1, ADOP2]
Export["../pics/invasionE_Zd.pdf", AZ0]
```

Out[∘]= {Ntot → 0.4 KK}

Out[∘]=



Out[∘]= ../pics/invasionE_Zd.pdf

Legend:
WT - only equilibrium is locally stable in the region left of the blue curve,
Drive - only equilibrium is locally stable in the region right of the red curve.

Purple: both equilibria are locally stable; bistability
White: none of the two boundary equilibria is locally stable: coexistence of WT and drive

Interior equilibrium for the parameters tested in R

In[∘]:=
```
neqs =
  Flatten[Evaluate[{DdGNtot == 0, DdGpD == 0, DdGHW == 0} /. demog /. ADO /. prms /.
      KK → 25 000 /. h0D → 1 /. cDrive → 0.6]];
NSolve[neqs, {Ntot, pD, dHW}, Reals]
```

Out[∘]= {{dHW → 0, pD → 1., Ntot → -2500.},
 {dHW → -0.0330327, pD → 0.195531, Ntot → 5796.09}, {dHW → 0, pD → 0, Ntot → 10 000.}}

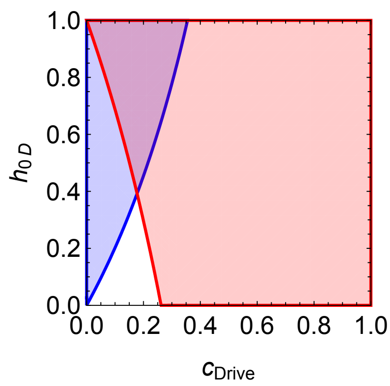
Fecundity only

Here we assume that only fecundity is affected by the drive

In[∘]:=
```
FO = {ω00 → 1, ω0D → 1, ωDD → 1, d00 → 0.6,
   d0D → 0.6, dDD → 0.6, β0D → β00 (1 - h0D) + βDD h0D};
```

In[•]:=
```
prms = {β00 → 1, βDD → 0.738};
FOP1 = RegionPlot[(EV0 /. FO /. prms) < 0,
  {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
  PlotStyle → RGBColor[0, 0, 1, 0.2], BoundaryStyle → Blue];
FOP2 = RegionPlot[Evaluate[(EVD1 /. FO /. prms)] < 0 &&
    Evaluate[(EVD2 /. FO /. prms)] < 0,
   {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
   PlotStyle → RGBColor[1, 0, 0, 0.2], BoundaryStyle → Red];
FZ0 = InvasionPic[FOP1, FOP2]
Export["../pics/invasionE_Zbeta.pdf", FZ0]
```

Out[•]=



Out[•]=
```
../pics/invasionE_Zbeta.pdf
```

Legend : see above (Adult death only)

Interior equilibrium for the parameters tested in R

In[•]:=
```
neqs = Flatten[Evaluate[
    {DdGNtot == 0, DdGpD == 0, DdGHW == 0} /. demog /. FO /. prms /. KK → 25 000 /.
      h0D → 1 /. cDrive → 0.3]];
NSolve[neqs, {Ntot, pD, dHW}, Reals]
```

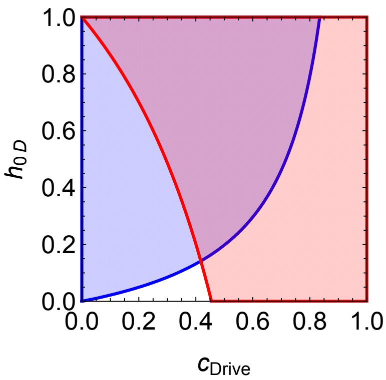
Out[•]=
```
{{dHW → 0, pD → 1., Ntot → -2540.93}, {dHW → 0, pD → 0.154962, Ntot → 7472.42},
 {dHW → 0, pD → 0, Ntot → 10 000.}, {Ntot → 10 000., pD → 0, dHW → 0}}
```

Zygote viability only

In[•]:=
```
ZO = {d00 → 0.6, d0D → 0.6, dDD → 0.6,
   β00 → 1, β0D → 1, βDD → 1, ω0D → ω00 (1 - h0D) + ωDD h0D};
```

In[•]:=
```
prms = {ω00 → 1, ωDD → 0.545};
ZOP1 = RegionPlot[(EV0 /. ZO /. prms) < 0,
   {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
   PlotStyle → RGBColor[0, 0, 1, 0.2], BoundaryStyle → Blue];
ZOP2 = RegionPlot[Evaluate[(EVD1 /. ZO /. prms)] < 0 &&
    Evaluate[(EVD2 /. ZO /. prms)] < 0,
   {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
   PlotStyle → RGBColor[1, 0, 0, 0.2], BoundaryStyle → Red];
OmegaZ0 = InvasionPic[ZOP1, ZOP2]
Export["../pics/invasionE_Zomega.pdf", OmegaZ0]
```

Out[•]=



Out[•]= ../pics/invasionE_Zomega.pdf

Legend : see above (Adult death only)

Interior equilibrium for the parameters tested in R

In[•]:=
```
neqs = Flatten[Evaluate[
    {DdGNtot == 0, DdGpD == 0, DdGHW == 0} /. demog /. ZO /. prms /. KK → 25 000 /.
      h0D → 1 /. cDrive → 0.5]];
NSolve[neqs, {Ntot, pD, dHW}, Reals]
```

Out[•]= {{dHW → 0, pD → 1., Ntot → -2522.94},
 {Ntot → 3819.56, pD → 0.308668, dHW → -0.0570609}, {dHW → 0, pD → 0, Ntot → 10 000.}}

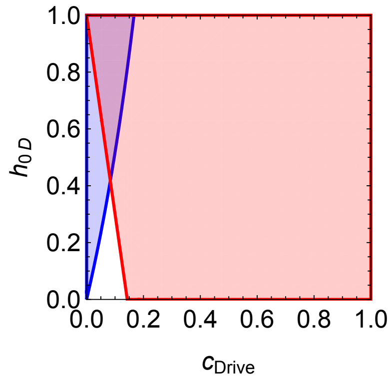
Plot the types of equilibria -- replacement drive

In[•]:=
```
EVD1 =.; EVD2 =.;
EVD1r = evZDr[[3]];
EVD2r = evZDr[[2]];
```

Adult death only

```
In[•]:= prms = {d00 → 0.6, dDD → 0.7};
ADOP1 = RegionPlot[Evaluate[(EV0 /. ADO /. prms)] < 0,
    {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
    PlotStyle → RGBColor[0, 0, 1, 0.2], BoundaryStyle → Blue];
ADOP2 = RegionPlot[Evaluate[(EVD1r /. ADO /. prms)] < 0 &&
     Evaluate[(EVD2r /. ADO /. prms)] < 0,
    {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
    PlotStyle → RGBColor[1, 0, 0, 0.2], BoundaryStyle → Red];
AZ0 = InvasionPic[ADOP1, ADOP2]
Export["../pics/invasionR_Zd.pdf", AZ0]
```

Out[•]=

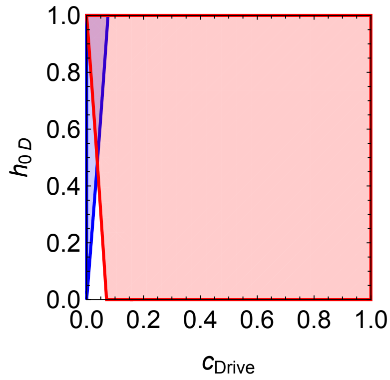



```
Out[•]= ../pics/invasionR_Zd.pdf
```

Fecundity only

In[•]:=
```
prms = {β00 → 1, βDD → 0.93};
FOP1 = RegionPlot[(EV0 /. FO /. prms) < 0,
   {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
   PlotStyle → RGBColor[0, 0, 1, 0.2], BoundaryStyle → Blue];
FOP2 = RegionPlot[Evaluate[(EVD1r /. FO /. prms)] < 0 &&
     Evaluate[(EVD2r /. FO /. prms)] < 0,
    {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
    PlotStyle → RGBColor[1, 0, 0, 0.2], BoundaryStyle → Red];
FZ0 = InvasionPic[FOP1, FOP2]
Export["../pics/invasionR_Zbeta.pdf", FZ0]
```

Out[•]=

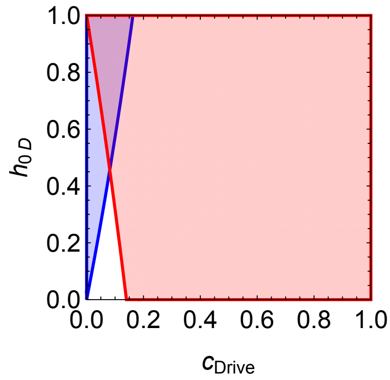


Out[•]= ../pics/invasionR_Zbeta.pdf

Zygote viability only

In[•]:=
```
prms = {ω00 → 1, ωDD → 0.86};
ZOP1 = RegionPlot[(EV0 /. ZO /. prms) < 0,
   {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
   PlotStyle → RGBColor[0, 0, 1, 0.2], BoundaryStyle → Blue];
ZOP2 = RegionPlot[Evaluate[(EVD1r /. ZO /. prms)] < 0 &&
    Evaluate[(EVD2r /. ZO /. prms)] < 0,
   {cDrive, 0, 1}, {h0D, 0, 1}, PlotRangePadding → None,
   PlotStyle → RGBColor[1, 0, 0, 0.2], BoundaryStyle → Red];
OmegaZ0 = InvasionPic[ZOP1, ZOP2]
Export["../pics/invasionR_Zomega.pdf", OmegaZ0]
```

Out[•]=



Out[•]= ../pics/invasionR_Zomega.pdf

Model with Brake

Model definition

Original variables

Zygote conversion

- Create gamete vector,
- Compute the different crossings;
- From this, deduce the different fertilized eggs (0D=D0);
- Let gene conversion take place

Abundance of the different genotypes right after fusion of the gametes

```
Vegg = ( {scaled}.Cross00.scaled
         {scaled}.Cross0D.scaled
         {scaled}.CrossDD.scaled
         {scaled}.Cross0B.scaled
         {scaled}.CrossDB.scaled
         {scaled}.CrossBB.scaled ) / (Ntot)^2 // FullSimplify;
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \left(\frac{(2\, n00\, \beta00+n0B\, \beta0B+n0D\, \beta0D)^2}{4\, Ntot^2}\right) \\ \left(\frac{(2\, n00\, \beta00+n0B\, \beta0B+n0D\, \beta0D)\,(n0D\, \beta0D+nDB\, \beta DB+2\, nDD\, \beta DD)}{2\, Ntot^2}\right) \\ \left(\frac{(n0D\, \beta0D+nDB\, \beta DB+2\, nDD\, \beta DD)^2}{4\, Ntot^2}\right) \\ \left(\frac{(2\, n00\, \beta00+n0B\, \beta0B+n0D\, \beta0D)\,(n0B\, \beta0B+2\, nBB\, \beta BB+nDB\, \beta DB)}{2\, Ntot^2}\right) \\ \left(\frac{(n0B\, \beta0B+2\, nBB\, \beta BB+nDB\, \beta DB)\,(n0D\, \beta0D+nDB\, \beta DB+2\, nDD\, \beta DD)}{2\, Ntot^2}\right) \\ \left(\frac{(n0B\, \beta0B+2\, nBB\, \beta BB+nDB\, \beta DB)^2}{4\, Ntot^2}\right) \end{pmatrix}$$

Gene conversion in the zygote

In[∘]:=
```
NVG = Conversion.Vegg // Flatten;
NVG // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{(2\, n00\, \beta00+n0B\, \beta0B+n0D\, \beta0D)^2}{4\, Ntot^2} \\ \frac{(2\, n00\, \beta00+n0B\, \beta0B+n0D\, \beta0D)\,(n0D\, \beta0D+nDB\, \beta DB+2\, nDD\, \beta DD)\,(1-c_{Drive})}{2\, Ntot^2} \\ \frac{(n0D\, \beta0D+nDB\, \beta DB+2\, nDD\, \beta DD)^2}{4\, Ntot^2} + \frac{(2\, n00\, \beta00+n0B\, \beta0B+n0D\, \beta0D)\,(n0D\, \beta0D+nDB\, \beta DB+2\, nDD\, \beta DD)\, c_{Drive}}{2\, Ntot^2} \\ \frac{(2\, n00\, \beta00+n0B\, \beta0B+n0D\, \beta0D)\,(n0B\, \beta0B+2\, nBB\, \beta BB+nDB\, \beta DB)}{2\, Ntot^2} \\ \frac{(n0B\, \beta0B+2\, nBB\, \beta BB+nDB\, \beta DB)\,(n0D\, \beta0D+nDB\, \beta DB+2\, nDD\, \beta DD)\,(1-c_{Brake})}{2\, Ntot^2} \\ \frac{(n0B\, \beta0B+2\, nBB\, \beta BB+nDB\, \beta DB)^2}{4\, Ntot^2} + \frac{(n0B\, \beta0B+2\, nBB\, \beta BB+nDB\, \beta DB)\,(n0D\, \beta0D+nDB\, \beta DB+2\, nDD\, \beta DD)\, c_{Brake}}{2\, Ntot^2} \end{pmatrix}$$

Dynamics of the different genotypes

In[∘]:=
```
dZn00 = NVG⟦1⟧ ω00 Birth[Ntot] - d00 Death[Ntot] n00;
dZn0D = NVG⟦2⟧ ω0D Birth[Ntot] - d0D Death[Ntot] n0D;
dZnDD = NVG⟦3⟧ ωDD Birth[Ntot] - dDD Death[Ntot] nDD;
dZn0B = NVG⟦4⟧ ω0B Birth[Ntot] - d0B Death[Ntot] n0B;
dZnDB = NVG⟦5⟧ ωDB Birth[Ntot] - dDB Death[Ntot] nDB;
dZnBB = NVG⟦6⟧ ωBB Birth[Ntot] - dBB Death[Ntot] nBB;
```

Export for C

The demographic functions are such that the death rate is constant and fecundity is density dependent

In[∘]:=

In[∘]:=
```
dZn00 /. chgvar // CForm
```

Out[∘]//CForm=
```
-(d00*pop(0)) + (omega00*(1 - popsize/K)*Power(2*beta00*pop(0) + beta0D*pop(1)
```

In[∘]:=
```
dZn0D /. chgvar // CForm
```

Out[∘]//CForm=
```
-(d0D*pop(1)) + ((1 - conversionD)*omega0D*(1 - popsize/K)*(2*beta00*pop(0) + l
    (2.*popsize)
```

In[*]:= `dZnDD /. chgvar // CForm`

Out[*]//CForm=
```
-(dDD*pop(2)) + omegaDD*popsize*(1 - popsize/K)*((conversionD*(2*beta00*pop(0)
        (beta0D*pop(1) + 2*betaDD*pop(2) + betaDB*pop(4)))/(2.*Power(popsize,2
      Power(beta0D*pop(1) + 2*betaDD*pop(2) + betaDB*pop(4),2)/(4.*Power(popsi2
```

In[*]:= `dZn0B /. chgvar // CForm`

Out[*]//CForm=
```
-(d0B*pop(3)) + (omega0B*(1 - popsize/K)*(2*beta00*pop(0) + beta0D*pop(1) + be
```

In[*]:= `dZnDB /. chgvar // CForm`

Out[*]//CForm=
```
-(dDB*pop(4)) + ((1 - conversionB)*omegaDB*(1 - popsize/K)*(beta0D*pop(1) + 2*b
    (2.*popsize)
```

In[*]:= `dZnBB /. chgvar // CForm`

Out[*]//CForm=
```
-(dBB*pop(5)) + omegaBB*popsize*(1 - popsize/K)*((conversionB*(beta0D*pop(1) +
        (beta0B*pop(3) + betaDB*pop(4) + 2*betaBB*pop(5)))/(2.*Power(popsize,2
      Power(beta0B*pop(3) + betaDB*pop(4) + 2*betaBB*pop(5),2)/(4.*Power(popsi2
```

Abundance of the different alleles (/2)

In[*]:=
```
dZn0 = dZn00 + 1/2 dZn0D + 1/2 dZn0D // FullSimplify;
dZnD = dZnDD + 1/2 dZn0D + 1/2 dZnDB // FullSimplify;
dZnB = dZnBB + 1/2 dZn0B + 1/2 dZnDB // FullSimplify;
```

New variables

Rewrite the dynamics with the new variables

Total population size

In[*]:= `dZNtot = FullSimplify[dZn0 + dZnD + dZnB] /. solCV3;`

Frequencies of Drive and Brake

In[*]:=
$$dZpD = \frac{dZnD}{Ntot} - \frac{\left(nDD + 1/2\ n0D + 1/2\ n0B\right)\ dZNtot}{Ntot^2}\ /.\ solCV3\ ;$$

$$dZpB = \frac{dZnB}{Ntot} - \frac{\left(nBB + 1/2\ n0B + 1/2\ nDB\right)\ dZNtot}{Ntot^2}\ /.\ solCV3\ ;$$

Deviation from Hardy - Weinberg

In[*]:=
$$dZHW0D = \frac{-n0D\ dZNtot + Ntot\ \left(dZn0D + 2\ Ntot\ \left(pD\ dZpB + \left(-1 + pB + 2\ pD\right)\ dZpD\right)\right)}{Ntot^2}\ /.\ solCV3;$$

$$dZHWDB = \frac{Ntot\ dZnDB - nDB\ dZNtot}{Ntot^2} - 2\ \left(pD\ dZpB + pB\ dZpD\right)\ /.\ solCV3;$$

$$dZHW0B = \frac{-n0B\ dZNtot + Ntot\ \left(dZn0B + 2\ Ntot\ \left(pB\ dZpD + \left(-1 + pD + 2\ pB\right)\ dZpB\right)\right)}{Ntot^2}\ /.\ solCV3;$$

Equilibrium stability

Stability of the WT only equilibrium

Jacobian matrix

In[•]:= JacZ =

```
( D[dZNtot, Ntot]  D[dZNtot, pD]  D[dZNtot, pB]  D[dZNtot, diffHW0D]  D[dZNtot, diffH
  D[dZpD, Ntot]    D[dZpD, pD]    D[dZpD, pB]    D[dZpD, diffHW0D]    D[dZpD, diffHW
  D[dZpB, Ntot]    D[dZpB, pD]    D[dZpB, pB]    D[dZpB, diffHW0D]    D[dZpB, diffHW
  D[dZHW0D, Ntot]  D[dZHW0D, pD]  D[dZHW0D, pB]  D[dZHW0D, diffHW0D]  D[dZHW0D, diffH
  D[dZHW0B, Ntot]  D[dZHW0B, pD]  D[dZHW0B, pB]  D[dZHW0B, diffHW0D]  D[dZHW0B, diffH
  D[dZHWDB, Ntot]  D[dZHWDB, pD]  D[dZHWDB, pB]  D[dZHWDB, diffHW0D]  D[dZHWDB, diffH
```

Out[•]= { ⋯ 1 ⋯ }

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WT-only stability -- check that the equilibrium is correct

In[•]:=
```
{dZNtot, dZpD, dZpB, dZHW0D, dZHW0B, dZHWDB} /. demog /.
   {pD → 0, pB → 0, diffHW0B → 0, diffHW0D → 0, diffHWDB → 0} /. sol0 // AF
```

Out[•]= {0, 0, 0, 0, 0, 0}

Eigenvalues

In[•]:=
```
evZWT = Eigenvalues[AF[JacZ /. demog /.
       {pD → 0, pB → 0, diffHW0B → 0, diffHW0D → 0, diffHWDB → 0} /. sol0]] // AF
```

Out[•]=
$$\left\{-\mathrm{dBB},\ -\mathrm{dDB},\ \mathrm{d00}-\beta 00^2\,\omega 00,\ -\mathrm{d0B}+\frac{\mathrm{d00}\,\beta 0\mathrm{B}\,\omega 0\mathrm{B}}{\beta 00\,\omega 00},\right.$$
$$-\frac{1}{2\,\beta 00\,\omega 00}\Big(\mathrm{d0D}\,\beta 00\,\omega 00+\mathrm{dDD}\,\beta 00\,\omega 00-\mathrm{d00}\,\beta 0\mathrm{D}\,\omega 0\mathrm{D}+\mathrm{d00}\,\beta 0\mathrm{D}\,\omega 0\mathrm{D}\,c_{\mathrm{Drive}}-2\,\mathrm{d00}\,\beta\mathrm{DD}\,\omega\mathrm{DD}\,c_{\mathrm{Drive}}+\sqrt{\Big(\big((\mathrm{d0D}+\mathrm{dDD})\,\beta 00\,\omega 00-\mathrm{d00}\,\beta 0\mathrm{D}\,\omega 0\mathrm{D}+\mathrm{d00}\,(\beta 0\mathrm{D}\,\omega 0\mathrm{D}-2\,\beta\mathrm{DD}\,\omega\mathrm{DD})\,c_{\mathrm{Drive}}\big)^2+4\,\beta 00\,\omega 00\,\big(-\mathrm{d0D}\,\mathrm{dDD}\,\beta 00\,\omega 00+\mathrm{d00}\,\mathrm{dDD}\,\beta 0\mathrm{D}\,\omega 0\mathrm{D}+\mathrm{d00}\,(-\mathrm{dDD}\,\beta 0\mathrm{D}\,\omega 0\mathrm{D}+2\,\mathrm{d0D}\,\beta\mathrm{DD}\,\omega\mathrm{DD})\,c_{\mathrm{Drive}}\big)\Big)}\Big),$$
$$\frac{1}{2\,\beta 00\,\omega 00}\Big(-\mathrm{d0D}\,\beta 00\,\omega 00-\mathrm{dDD}\,\beta 00\,\omega 00+\mathrm{d00}\,\beta 0\mathrm{D}\,\omega 0\mathrm{D}-\mathrm{d00}\,\beta 0\mathrm{D}\,\omega 0\mathrm{D}\,c_{\mathrm{Drive}}+2\,\mathrm{d00}\,\beta\mathrm{DD}\,\omega\mathrm{DD}\,c_{\mathrm{Drive}}+\sqrt{\Big(\big((\mathrm{d0D}+\mathrm{dDD})\,\beta 00\,\omega 00-\mathrm{d00}\,\beta 0\mathrm{D}\,\omega 0\mathrm{D}+\mathrm{d00}\,(\beta 0\mathrm{D}\,\omega 0\mathrm{D}-2\,\beta\mathrm{DD}\,\omega\mathrm{DD})\,c_{\mathrm{Drive}}\big)^2+4\,\beta 00\,\omega 00\,\big(-\mathrm{d0D}\,\mathrm{dDD}\,\beta 00\,\omega 00+\mathrm{d00}\,\mathrm{dDD}\,\beta 0\mathrm{D}\,\omega 0\mathrm{D}+\mathrm{d00}\,(-\mathrm{dDD}\,\beta 0\mathrm{D}\,\omega 0\mathrm{D}+2\,\mathrm{d0D}\,\beta\mathrm{DD}\,\omega\mathrm{DD})\,c_{\mathrm{Drive}}\big)\Big)}\Big)\left.\right\}$$

Comparison to the value that was previously found -> same

In[•]:=
```
evZWT⟦6⟧ - evZ0⟦3⟧ // AF
```

Out[•]= 0