File S1: Details on MAGBLUP-RI derivation

Let us consider the following model for genetic values of admixed individuals presented in Eq. 6:

$$G_i = \sum_{m=1}^{M} \left[A_{imA} \left(\beta^0_{mA} + W_{im}(\beta^1_{mA} - \beta^0_{mA}) \right) + A_{imB} \left(\beta^0_{mB} + W_{im}(\beta^1_{mB} - \beta^0_{mB}) \right) \right]$$

To derive the MAGBLUP-RI Gaussian variance component model, the following steps were followed: (i) calculate the expected genetic value following the MAGBLUP-RI modeling described in the "Materials and methods" section, (ii) derive an equivalent statistical model for genetic values centered on their expected value, and (iii) calculate the covariance between genetic values using the equivalent model.

1 - Expected Genetic value

$$\mathrm{E}(G_i|\pi_i) = \pi_i \sum_{m=1}^{M} \left(\beta^0_{mA} + f_{mA}(\beta^1_{mA} - \beta^0_{mA}) \right) + (1-\pi_i) \sum_{m=1}^{M} \left(\beta^0_{mB} + f_{mB}(\beta^1_{mB} - \beta^0_{mB}) \right)$$

$$\mathrm{E}(G_i|\pi_i) = \pi_i \sum_{m=1}^{M} \mu_{mA} + (1-\pi_i) \sum_{m=1}^{M} \mu_{mB}$$

$$\mathrm{E}(G_i|\pi_i) = \pi_i \mu_A + (1-\pi_i)\mu_B$$

2 - Equivalent model

An equivalent model for genetic values is:

$$G_i|\pi_i = \pi_i\mu_A + (1-\pi_i)\mu_B + \sum_{m=1}^{M} A^*_{im} \left(\mu_{mA} - \mu_{mB} \right) + \sum_{m=1}^{M} W^*_{imA} \left(\beta^1_{mA} - \beta^0_{mA} \right) + \sum_{m=1}^{M} W^*_{imB} \left(\beta^1_{mB} - \beta^0_{mB} \right)$$

with

- $A^*_{im} = A_{imA} - \pi_i$: centered allele ancestry of individual i at locus m
- $\mathrm{Cov}\left(A^*_{im}, A^*_{jm}\right) = \theta^A_{ij} - \pi_i\pi_j = \Delta_{ij}$
- $\mathrm{Cov}\left(A^*_{im}, A^*_{jm'}\right) = -\dfrac{\Delta_{ij}}{M-1}$ (see derivation below)
- $W^*_{imA} = A_{imA}(W_{im} - f_{mA})$: centered allele genotype of individual i for allele originated from group A at locus m
- $\mathrm{Cov}\left(W^*_{imA}, W^*_{jmA}\right) = \theta^A_{ij}\alpha^A_{ij} f_{mA}(1 - f_{mA})$
- $\mathrm{Cov}\left(W^*_{imA}, W^*_{jm'A}\right) = 0$
- $W^*_{imB} = A_{imB}(W_{im} - f_{mB})$: centered allele genotype of individual i for allele originated from group B at locus m
- $\mathrm{Cov}\left(W^*_{imB}, W^*_{jmB}\right) = \theta^B_{ij}\alpha^B_{ij} f_{mB}(1 - f_{mB})$
- $\mathrm{Cov}\left(W^*_{imB}, W^*_{jm'B}\right) = 0$
- $A^*_{im} \perp\!\!\!\perp W^*_{im'A} \perp\!\!\!\perp W^*_{im''B}$ for all m, m', m'' due to centering of variables

3 - Covariance between genetic values

$$\begin{aligned}
\mathrm{Cov}(G_i, G_j|\pi_i, \pi_j, \theta_{ij}^A, \theta_{ij}^B, \alpha_{ij}^A, \alpha_{ij}^B) &= \sum_{m=1}^{M}\sum_{m'=1}^{M} \mathrm{Cov}\left(A_{im}^*, A_{jm'}^*\right)\left(\mu_{mA} - \mu_{mB}\right)\left(\mu_{m'A} - \mu_{m'B}\right) \\
&\quad + \sum_{m=1}^{M}\sum_{m'=1}^{M} \mathrm{Cov}\left(W_{imA}^*, W_{jm'A}^*\right)\left(\beta_{mA}^1 - \beta_{mA}^0\right)\left(\beta_{m'A}^1 - \beta_{m'A}^0\right) \\
&\quad + \sum_{m=1}^{M}\sum_{m'=1}^{M} \mathrm{Cov}\left(W_{imB}^*, W_{jm'B}^*\right)\left(\beta_{mB}^1 - \beta_{mB}^0\right)\left(\beta_{m'B}^1 - \beta_{m'B}^0\right) \\
\mathrm{Cov}(G_i, G_j|\pi_i, \pi_j, \theta_{ij}^A, \theta_{ij}^B, \alpha_{ij}^A, \alpha_{ij}^B) &= \sum_{m=1}^{M} \mathrm{Cov}\left(A_{im}^*, A_{jm}^*\right)\left(\mu_{mA} - \mu_{mB}\right)^2 \\
&\quad + \sum_{m=1}^{M}\sum_{m'\neq m}^{M} \mathrm{Cov}\left(A_{im}^*, A_{jm'}^*\right)\left(\mu_{mA} - \mu_{mB}\right)\left(\mu_{m'A} - \mu_{m'B}\right) \\
&\quad + \sum_{m=1}^{M} \mathrm{Cov}\left(W_{imA}^*, W_{jmA}^*\right)\left(\beta_{mA}^1 - \beta_{mA}^0\right)^2 \\
&\quad + \sum_{m=1}^{M} \mathrm{Cov}\left(W_{imB}^*, W_{jmB}^*\right)\left(\beta_{mB}^1 - \beta_{mB}^0\right)^2 \\
\mathrm{Cov}(G_i, G_j|\pi_i, \pi_j, \theta_{ij}^A, \theta_{ij}^B, \alpha_{ij}^A, \alpha_{ij}^B) &= \Delta_{ij}\sum_{m=1}^{M}\left(\mu_{mA} - \mu_{mB}\right)^2 \\
&\quad - \frac{\Delta_{ij}}{M-1}\sum_{m=1}^{M}\sum_{m'\neq m}^{M}\left(\mu_{mA} - \mu_{mB}\right)\left(\mu_{m'A} - \mu_{m'B}\right) \\
&\quad + \theta_{ij}^A\alpha_{ij}^A\sum_{m=1}^{M} f_{mA}(1-f_{mA})\left(\beta_{mA}^1 - \beta_{mA}^0\right)^2 \\
&\quad + \theta_{ij}^B\alpha_{ij}^B\sum_{m=1}^{M} f_{mB}(1-f_{mB})\left(\beta_{mB}^1 - \beta_{mB}^0\right)^2 \\
\mathrm{Cov}(G_i, G_j|\pi_i, \pi_j, \theta_{ij}^A, \theta_{ij}^B, \alpha_{ij}^A, \alpha_{ij}^B) &= \Delta_{ij}\sum_{m=1}^{M}\left(\mu_{mA} - \mu_{mB}\right)^2 + \frac{\Delta_{ij}}{M-1}\sum_{m=1}^{M}\left(\mu_{mA} - \mu_{mB}\right)^2 \\
&\quad - \frac{\Delta_{ij}}{M-1}\sum_{m=1}^{M}\sum_{m'=1}^{M}\left(\mu_{mA} - \mu_{mB}\right)\left(\mu_{m'A} - \mu_{m'B}\right) \\
&\quad + \theta_{ij}^A\alpha_{ij}^A\sigma_{G_A}^2 + \theta_{ij}^B\alpha_{ij}^B\sigma_{G_B}^2 \\
\mathrm{Cov}(G_i, G_j|\pi_i, \pi_j, \theta_{ij}^A, \theta_{ij}^B, \alpha_{ij}^A, \alpha_{ij}^B) &= \Delta_{ij}\left[\frac{M}{M-1}\sum_{m=1}^{M}\left(\mu_{mA} - \mu_{mB}\right)^2 - \frac{1}{M-1}\left(\sum_{m=1}^{M}\left(\mu_{mA} - \mu_{mB}\right)\right)^2\right] \\
&\quad + \theta_{ij}^A\alpha_{ij}^A\sigma_{G_A}^2 + \theta_{ij}^B\alpha_{ij}^B\sigma_{G_B}^2 \\
\mathrm{Cov}(G_i, G_j|\pi_i, \pi_j, \theta_{ij}^A, \theta_{ij}^B, \alpha_{ij}^A, \alpha_{ij}^B) &= \Delta_{ij}\left[\frac{M}{M-1}\sum_{m=1}^{M}\left(\mu_{mA} - \mu_{mB}\right)^2 - \frac{1}{M-1}\left(\mu_{A} - \mu_{B}\right)^2\right] \\
&\quad + \theta_{ij}^A\alpha_{ij}^A\sigma_{G_A}^2 + \theta_{ij}^B\alpha_{ij}^B\sigma_{G_B}^2 \\
\mathrm{Cov}(G_i, G_j|\pi_i, \pi_j, \theta_{ij}^A, \theta_{ij}^B, \alpha_{ij}^A, \alpha_{ij}^B) &= \Delta_{ij}\sigma_S^2 + \theta_{ij}^A\alpha_{ij}^A\sigma_{G_A}^2 + \theta_{ij}^B\alpha_{ij}^B\sigma_{G_B}^2
\end{aligned}$$

Note that the segregation variance can be approximated by $\sigma_S^2 \approx \sum_{m=1}^{M}(\mu_{mA} - \mu_{mB})^2$ when the number of loci is large and the difference between group-specific means $\mu_A - \mu_B$ is small (i.e. segregation effects tend to cancel each other out over loci).

Derivation of $\mathrm{Cov}\left(A_{im}^*, A_{jm'}^*\right)$

$$
\begin{aligned}
\mathrm{Cov}\left(A_{im}^*, A_{jm'}^*\right) &= \mathrm{E}(A_{im}^* A_{jm'}^*) - \mathrm{E}(A_{im}^*)\mathrm{E}(A_{jm'}^*) \\
\mathrm{Cov}\left(A_{im}^*, A_{jm'}^*\right) &= \mathrm{E}(A_{imA} A_{jm'A}) - \pi_i \pi_j + 0 \\
\mathrm{Cov}\left(A_{im}^*, A_{jm'}^*\right) &= \mathrm{P}\left(A_{imA} = 1 \cap A_{jm'A} = 1\right) - \pi_i \pi_j \\
\mathrm{Cov}\left(A_{im}^*, A_{jm'}^*\right) &= \mathrm{P}\left(A_{imA} = 1 | A_{jm'A} = 1\right) \mathrm{P}\left(A_{jm'A} = 1\right) - \pi_i \pi_j \\
\mathrm{Cov}\left(A_{im}^*, A_{jm'}^*\right) &= \left[\mathrm{P}\left(A_{imA} = 1 \cap A_{im'A} = 0 | A_{jm'A} = 1\right)\right. \\
&\qquad \left. + \mathrm{P}\left(A_{imA} = 1 \cap A_{im'A} = 1 | A_{jm'A} = 1\right)\right] \pi_j - \pi_i \pi_j \\
\mathrm{Cov}\left(A_{im}^*, A_{jm'}^*\right) &= \left[\mathrm{P}\left(A_{imA} = 1 | A_{im'A} = 0, A_{jm'A} = 1\right) \mathrm{P}\left(A_{im'A} = 0 | A_{jm'A} = 1\right)\right. \\
&\qquad \left. + \mathrm{P}\left(A_{imA} = 1 | A_{im'A} = 1, A_{jm'A} = 1\right) \mathrm{P}\left(A_{im'A} = 1 | A_{jm'A} = 1\right)\right] \pi_j - \pi_i \pi_j \\
\mathrm{Cov}\left(A_{im}^*, A_{jm'}^*\right) &= \left(\frac{\pi_i M}{M-1} \times \frac{\left(\pi_j - \theta_{ij}^A\right) M}{\pi_j M} + \frac{\pi_i M - 1}{M-1} \times \frac{\theta_{ij}^A M}{\pi_j M}\right) \pi_j - \pi_i \pi_j \\
\mathrm{Cov}\left(A_{im}^*, A_{jm'}^*\right) &= \frac{M}{M-1} \pi_i \pi_j - \frac{1}{M-1} \theta_{ij}^A - \pi_i \pi_j \\
\mathrm{Cov}\left(A_{im}^*, A_{jm'}^*\right) &= -\frac{\Delta_{ij}}{M-1}
\end{aligned}
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